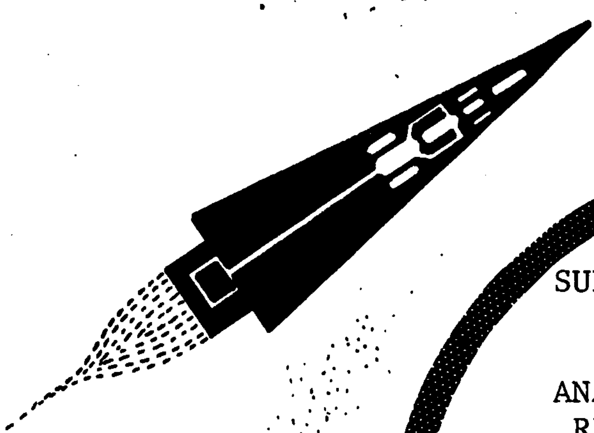
SPACE POWER AND PROPULSION SECTION



SUPPLEMENTARY REPORT #1

ANALYTICAL METHODS FOR
RESISTANCE JET DESIGN

CONTRACT NO. NAS 5-9013

AUGUST, 1964

Prepared for:

Goddard Space Flight Center
Greenbelt, Maryland

MISSILE and SPACE DIVISION

GENERAL ELECTRIC

CINCINNATI, OHIO

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Submitted by:

Robert Viventi

Robert Viventi, Project Engineer

ELECTRICAL SPACE PROPULSION PROJECT
SPACE POWER AND PROPULSION SECTION
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PREFACE

This report was prepared by the Space Power and Propulsion Section of the General Electric Company in Evendale, Ohio under NASA Contract NAS 5-9013, "The Development of Resistance Jet Thruster Systems". The work is administered under the direction of the Goddard Space Flight Center with Mr. James Bridger as Project Engineer.

Some of the analyses presented herein have previously been presented in the references, whereas some are being reported for the first time. Most of this work is the effort of the following contributors: R. Richter, M. L. Bromberg, R. E. Viventi, H. Brown and L. L. Cumbers.

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ABSTRACT

Analytical methods are presented for designing a resistance jet thruster to be used for station-keeping and attitude control functions of a space vehicle. The steps considered include the thermodynamic design, optimization of the expansion nozzle, length of flow passage, size and life of the heater filament, and radiation heat shielding. The transient performance of the thruster is then calculated, and appropriate modifications can be made if desired, to change the time constant and minimum impulse bit of the thruster. Effects on the vehicle are analyzed; and a procedure is given for designing the thruster so as to minimize the overall weight penalty on the vehicle.

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NOMENCLATURE

A	Area, in^2
A_1^*	Area of sonic inlet orifice, in^2
A_3^*	Area of sonic engine throat, in^2
A_e	Exit area, in^2
C_g	Specific heat of gas, B/lb.°R
C_o	Sonic velocity at stagnation temperature, ft/sec
C_p	Specific heat, B/lb.°R
C_T	Specific heat of thruster, B/lb°R
E, ϵ_{eff}	Effective emissity, dimensionless
F	Steady-state thrust, lb.
g	Acceleration of gravity, 32.2 ft/sec^2
h	Heat transfer coefficient, B/ft.^{2} hr.°R
I, I_T	Impulse, lb-sec
I	Current, amps
I_{sp}	Specific impulse, sec
K_1	Constant defined by equation (42)
K_i	Constant given in equation (38)
k	Thermal conductivity, B/hr.ft.°R; Ratio of specific heats
L	Length of flow passage or length of heater wire, in.
N	Number of thrusters; Number of revolutions
n	Number of layers of heat shielding
P	Electrical power, watts; Pressure, psia
P_e	Exit pressure, psia
Q_T	Total heat transferred to the propellant, watts
q	Heat transferred, watts
R	Resistance of heat wire, ohms; Gas constant, ft/°R

Nomenclature, Cont'd

R_M	Mean radius of flow annulus, in.
r	Radius, in
T,t	Temperature, ^oR
t	Time, sec
t_V	Time inlet valve is open, sec.
V	Propellant velocity, in/sec; Volume, ft^3; Voltage, volts
ΔV	Characteristic velocity, ft/sec
$\dot{W}$	Mass flow rate, lb/sec
W_1	Weight of vehicle without propellant, lb
W_c	Weight of gas in chamber, lb.
W_o	Total weight of vehicle, lb.
W_{PP}	Weight of propellant, lb.
W_s	Total weight of resistance jet system, lb.
W_T	Mass of thruster, lb.
w	Ratio of power supply weight to power, lb/watt
w_T	Ratio of tankage to propellant weight
α	Ratio of sonic flow areas, A_1^*/A_3^*
$1/\alpha^2$	Thermal diffusivity, in^2/sec
δ	Width of flow passage, in.
ϵ	Emissivity, dimensionless
θ_1	Time at which flow is stopped, sec
μ	Viscosity, lb/ft. sec
ξ	Ratio of chamber gas temperature to supply temperature, T_c/T_o
ρ	Propellant density, lb/ft^3
σ	Stefan-Boltzman constant, $5.668 \times 10^{-12} \frac{watts}{cm^2 {}^oR^4}$; Constant defined by equation (28)
τ	Time constant, sec

I. INTRODUCTION

This report is a compilation of analyses which are useful in the design of resistance jet thrusters. These analyses have previously been used in determining design parameters for actual engines, and examples of applying the analyses are included throughout.

The design procedure given in this report involves first specifying a thrust level and then proceding step-by-step to a complete thruster design. In actual practice, these steps are not, in fact, done consecutively but often concurrently. At many times during the design of a resistance jet, practical considerations will necessitate changes and compromises not accounted for in this procedure. However, this report provides the necessary analyses for taking into account the important parameters in resistance jet design.

The thermodynamic design of the thruster itself is the first consideration, and the expansion nozzle is designed to minimize the boundary layer losses. Then the physical design of the thruster body is obtained by finding the required length of flow passage and the amount of heater wire needed. Since the engine lifetime is normally dependent on the heater life, the prediction of operating life can then be made. Finally, a calculation is made of the amount of heat shielding required to maintain radiation losses below a specified amount.

A resistance jet used for station-keeping and attitude control of a space vehicle is usually expected to operate repeatedly with short pulses of thrust. Therefore it is necessary to know about the transient performance of the thruster.

However, the design procedure so far has been based completely on the steady-state performance goals. Accordingly, it is essential to determine the transient effects of the design parameters to make sure they are compatible with the particular thruster application.

Finally, it is important to consider the effect of the thruster on the vehicle. This can be done at the beginning of the design or, in the case when the performance goals are previously specified, at the conclusion of the design. In any case, it is advisable to compare the thruster design with the optimum design based on vehicle specifications. This can be done with the optimization analysis given in this report.

In the Appendix is given the output of the nozzle optimization program for a particular design example. A list of references is included which contain several of these analyses in greater detail.

II. DESIGN PROCEDURE FOR A RESISTANCE JET THRUSTER

The resistance jet is an engine which can be used for a variety of applications. In this report, the particular application is to provide small amounts of thrust at varying intervals for station-keeping and attitude control of a satellite. For such an application, we can specify the thrust level required for the engine. Let us assume a thrust of 0.05 lb, which is a representative thrust level for combined station-keeping and attitude control functions.

A. Thermodynamic Design

The selection of propellant depends mostly upon the specific impulse desired; for this example we will consider hydrogen as the propellant.

Next, we shall assume an operating temperature and pressure for the propellant, based upon the materials used in the thruster. For this example, we can consider that the critical parts of the engine will be made of high temperature steel alloys, with boron nitride as an insulator. A safe operating temperature for these materials, which will still give a reasonable level of specific impulse, can be taken at $2700^{\circ}R$. The chamber pressure will be assumed to be 5 atmospheres.

Now we need thermodynamic tables for hydrogen. These have been generated by a computer program and, for hydrogen at 5 atmospheres and $2700^{\circ}R$, the ideal specific impulse is listed in Table 1 as a function of the ideal nozzle area ratio. If we take an area ratio of 100:1, the following values can be read:

TABLE I

THERMODYNAMIC AND FLUID DYNAMIC RELATICNS FOR HYDROGEN

STAGNATICN PRESSURE	5.0000+CO ATM	AMBIENT PRESSURE	0. MM HG
STAGNATICN TEMPERATURE	2.7000+C3 DEGREE R	AMBIENT TEMPERATURE	5.4000+02 DEGREE R

FLOW PER UNIT THROAT AREA	1.9621-C1 LB/SEC-IN2	BASE ENTHALPY	7.4308+03 B/LB
FLOW PER POWER	1.1451-C4 LB/SEC-KW	ENTHALPY	1.5710+04 B/LB
ENERGY ADDITICN PER POUND	8.2790+C3 B/LB	SPECIFIC HEAT AT TO	3.8301+00 B/LB-DEGREE R
FRACTICN DISSCCIATEC	4.4025-C6	SPECIFIC GAS CCNSTANT	9.8442-01 B/LB-DEGREE R
FRACTICN IONIZED	4.4721-16	POWER PER UNIT THROAT AREA	1.7134+03 KW/IN2
DENSITY AT TC	5.1173-C3 LB/FT3		

SPECIFIC IMPULSE SEC	PRESSURE RATIO	AREA RATIO	THERMODYNAMIC EFFICIENCY	POWER/ THRUST KW/LB	FREE STREAM TEMPERATURE DEGREE R	FREE STREAM MACH NUMBER	SPECIFIC WEIGHT LB/FT3	THRUST/UNIT THROAT AREA LB/IN2	SPECIFIC HEAT B/LB-R
4.7280+02	1.8688+00	1.0000+00	5.5864-01	1.8470+C1	2.2940+03	1.0000+00	3.2228-03	9.2769+01	3.7280+00
5.4561+02	1.0386+01	2.0000+00	7.4397-01	1.6005+01	1.4384+03	2.1728+00	9.2482-04	1.0706+02	3.5290+00
5.7114+02	2.0551+01	3.0000+00	8.1519-01	1.5290+01	1.1875+03	2.6016+00	5.6617-04	1.1207+02	3.4816+00
5.9519+02	4.6341+01	5.0000+00	8.8531-01	1.4672+01	9.4229+02	3.1304+00	3.1641-04	1.1679+02	3.4438+00
6.1858+02	1.3372+02	1.0000+01	9.5627-01	1.4117+01	6.9509+02	3.8738+00	1.4865-04	1.2138+02	3.4167+00
6.3503+02	3.7510+02	2.0000+01	1.0078+00	1.3751+01	5.1610+02	4.6789+00	7.1370-05	1.2460+02	3.4057+00
6.4241+02	6.8002+02	3.0000+01	1.0313+00	1.3593+01	4.3453+02	5.1882+00	4.6758-05	1.2605+02	3.4034+00
6.4994+02	1.4298+03	5.0000+01	1.0557+00	1.3436+01	3.5048+02	5.8779+00	2.7572-05	1.2753+02	3.4030+00
6.5775+02	3.8863+03	1.0000+02	1.0812+00	1.3276+C1	2.6246+02	6.9134+00	1.3546-05	1.2906+02	3.4049+00
6.6353+02	1.0489+04	2.0000+02	1.1003+00	1.3161+01	1.9700+02	8.0832+00	6.6869-06	1.3019+02	3.4080+00
6.6895+02	3.8675+04	5.0000+02	1.1183+00	1.3054+01	1.3517+02	9.8757+00	2.6430-06	1.3126+02	3.4123+00

Specific impulse = 658 sec.

Flow/throat area = 0.196 lb/(sec-in^2)

Thrust/throat area = 129 lb/in^2

Power/thrust = 13.3 Kw/lb

Therefore we can calculate,

$$\text{Throat area} = \frac{0.05 \text{ lb}}{129 \text{ lb/in}^2} = 0.000388 \text{ in}^2$$

$$\text{Throat diameter} = \left(\frac{4(.000388)}{\pi}\right)^{1/2} = 0.0222 \text{ in.}$$

Exit area = 0.0388 in^2

Exit diameter = 0.222 in

$$\text{Power} = 13.3 \frac{\text{Kw}}{\text{lb}} \ (0.05 \text{ lb}) = 665 \text{ watts}$$

$$\text{Flow rate} = \frac{0.196 \text{ lb}}{\text{sec - in}^2} \times 0.000388 \text{ in}^2 = 7.6 \times 10^{-5} \ \frac{\text{lb}}{\text{sec}}$$

This procedure can therefore be used to establish the thermodynamic design of a resistance jet thruster, not considering the radiation and boundary layer losses.

B. Optimization of Nozzle Design

If the boundary layer losses are taken into consideration, the nozzle area ratio can be chosen so as to optimize the overall performance. The radiation losses are usually small and relatively constant so they have only a slight effect on the optimization. The boundary layer losses are found by a computer program which calculates the size of the boundary layer for different nozzle angles and lengths, and finds the heat transfer and fluid friction losses for each case. This program has been applied to the design of the example engine in this report. The output is shown in Appendix A which indicates that the optimum nozzle angle is 25° and that the thrust will decrease for an effective area ratio greater than 15.3. (The effective area ratio is the actual area ratio corrected for the boundary layer displacement thickness).

The results of this program show that an optimum nozzle design leads to the following parametric values:

Specific impulse = 596 sec.

Flow/throat area = 0.196 lb/(sec - in^2)

Flow rate = 7.6×10^{-5} lb/sec.

Throat diameter = 0.0222 inch

Nozzle angle = 25°

Actual area ratio = 38.2:1

Exit diameter = 0.137 in.

Actual thrust = 0.0453 lb.

Power = 665 watts

This optimization program has therefore determined the optimum design for the nozzle without affecting the thermodynamic design. It also indicates the boundary layer losses in the nozzle and the resulting values of effective specific impulse and thrust.

C. Calculation of Flow Path

To determine the length of path needed for the propellant, we refer to an analysis of the radiation and convection heating of a gas in an annulus with heated core (Reference 1). It shows that the temperature of the gas in the annulus will exponentially approach the temperature of the hot wall. The total heat transferred is given by:

$$Q_T = K_1 + K_2 \left[\left(1 - e^{-\left(\frac{\pi}{\delta\alpha}\right)^2 t}\right) + \left(\frac{1 - e^{-9\left(\frac{\pi}{\delta\alpha}\right)^2 t}}{9}\right) + \left(\frac{1 - e^{-25\left(\frac{\pi}{\delta\alpha}\right) t}}{25}\right) + \ldots\ldots\ldots \right] \qquad (1)$$

As t, the residence time, approaches infinity, this becomes:

$$Q_T = K_1 + K_2 \left[1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \ldots\ldots \right]$$

Therefore, if we choose a value of the exponent that makes the first expotential term have the value 0.01, this will give a total heat transfer which is within one percent of the maximum heat transfer obtainable. So we take

$$e^{-\left(\frac{\pi}{\delta\alpha}\right)^2 t} = 0.01 \qquad (2)$$

where t = time in the annulus, sec

δ = width of flow passage, in

α^2 = (thermal diffusivity)$^{-1}$, $\frac{\rho Cp}{k}$

Now the residence time is given by the length of passage divided by the velocity,

$$t = \frac{L}{V} = L \frac{\rho A}{\dot{W}} = L\rho \frac{(2\pi R_m \delta)}{\dot{W}} \tag{3}$$

Therefore the exponent of equation (2) becomes

$$-\frac{\pi^2}{\delta^2 \alpha^2} t = -\frac{\pi^2 k}{\delta^2 \rho C_p} \cdot \frac{L\rho 2\pi R_m \delta}{\dot{W}} = -\frac{2\pi^3 k L R_m}{\dot{W} \delta C_p} \tag{4}$$

and we can take the logarithm of both sides of equation (2)

$$\ln 0.01 = -4.605 = -\frac{2\pi^3 k L R_m}{\dot{W} \delta C_p} \tag{5}$$

Assuming typical values as follows, we can calculate the flow length:

$\delta = 0.020$ in

$T = 2700^\circ R$

$R_m = 0.25$ in

$k = 0.25\ ^B/\text{hr ft.} - {}^\circ R$ at $2000^\circ R$

$C_p = 0.35\ ^B/\text{lb-}{}^\circ R$ at $2000^\circ R$

$\dot{W} = 7.6 \times 10^{-5}$ lb/sec

Then $L = \frac{4.605\ (7.6 \times 10^{-5})(.02)(.35)(3600)}{2\ (3.14)^3\ (.25)\ (.25)} = 0.027$ in.

Therefore, a design of the assumed diameter and thickness which has a flow path longer than 0.027 inches is sufficient to achieve a propellant temperature which is essentially equal to the hot wall temperature. The design example given in this report has a flow path of 1.5 inches to provide sufficient room for the heater wire, and therefore the propellant will be at wall temperature throughout almost the entire flow passage.

D. Heater Wire Size and Lifetime

The preceding chapters have discussed the thermodynamic and thermal design of the resistance jet engine. In order to complete the mechanical design it is necessary to know the size and length of wire to be used for the heater. The life of the wire can also be predicted and this will then determine the engine lifetime. The design procedure of the heater wire is as follows:

From the thermodynamic design, we know the power input of the engine.

Power = 665 watts.

We also know the temperature of the gas.

Temperature = $2700^{\circ}R$.

and there is a temperature drop through the boron nitride and stainless steel surrounding the wire. Therefore, we can calculate the heater wire temperature for a typical engine design (see Fig. 1).

Density of propellant (at average temperature $1600^{\circ}R$) = $8.8 \frac{lb}{ft^3}$

Flow path: .020 inch annulus around a 1/2" diameter heater core.

Flow area: $\frac{\pi}{4}\left[(.54)^2 - (.5)^2\right] = 0.033 \ in^2$

Flow rate: 7.6×10^{-5} lb/sec

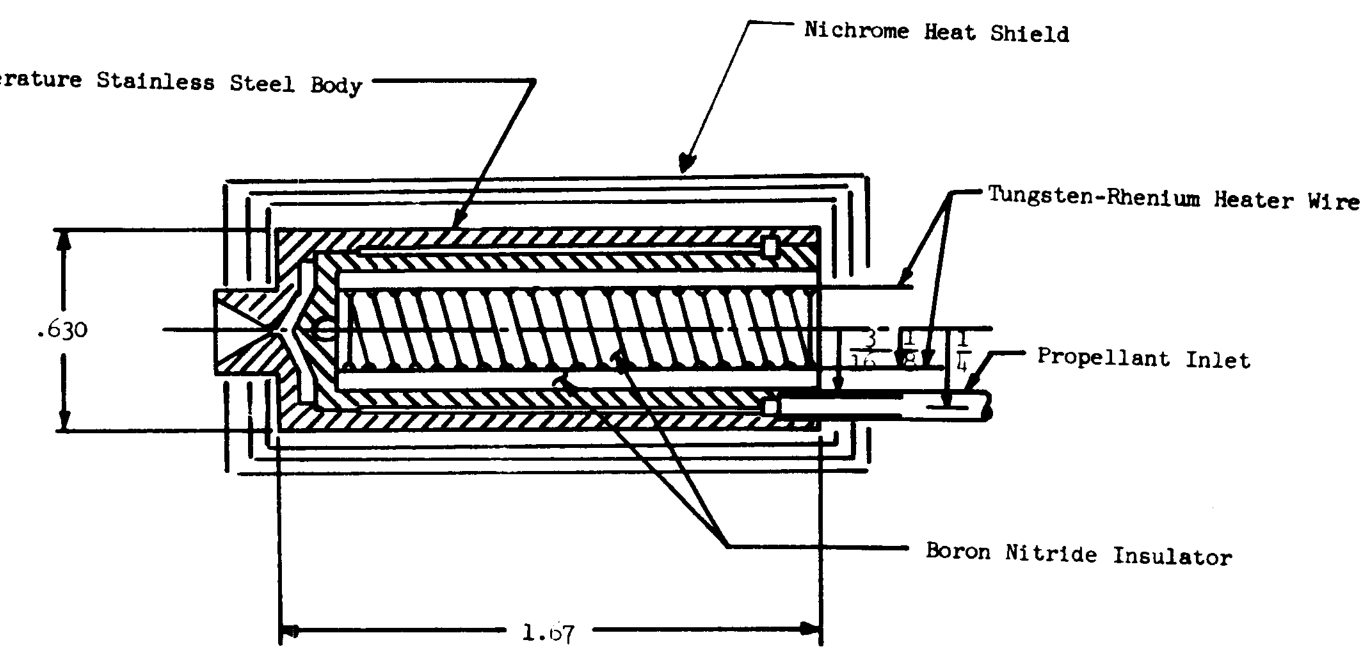


Fig. 1: Thermal Storage Resistance Thruster

Velocity of propellant: $= \dfrac{7.6 \times 10^{-5} \times 144}{8.8 \times 10^{-3} \times 0.033} = \dfrac{37.7 \text{ ft.}}{\text{sec}}$

Viscosity of propellant: (at $1600^{\circ}R$) = 1.2×10^{-5} lb/(sec-ft)

Reynolds number $= \dfrac{\rho V \delta}{\mu} = \dfrac{8.8 \times 10^{-3} \times 37.7 \times 1/16}{1.2 \times 10^{-5} \times 12} = 144$

This is therefore a case of laminar flow heat transfer to a gas in an annulus. From Jacob (Reference 2) the heat transfer coefficient for this case is:

$$h = \frac{2k}{\delta} = 2\left(0.2 \frac{B}{hrft^2\ ^{o}R}\right) \Big/ \left(\frac{.02}{12}\right) \text{ ft.} \tag{6}$$

$$h = 240 \ \frac{B}{hr.ft^2\ ^{o}R}$$

For heat transfer through a composite cylinder to a gas (see Fig. 1) the temperature drop is given by:

$$t_1 - t_4 = \frac{q}{L}\left[\frac{\ln r_2/r_1}{2\pi k_{12}} + \frac{\ln r_3/r_2}{2\pi k_{23}} + \frac{1}{2\pi r_3 h_{34}}\right] \tag{7}$$

$$= \frac{665 \text{ watts}}{1.5 \text{ in}}\left[\frac{\ln\left(\frac{3}{16}/\frac{2}{16}\right)}{2\pi(15)} + \frac{\ln\left(\frac{4}{16}/\frac{3}{16}\right)}{2\pi(10)} + \frac{1}{2\pi(\frac{1}{4})(\frac{1}{12})(240)}\right]$$

$$= \frac{665}{1.5} \times 12 \times 3.413 \left[\frac{\ln(3/2)}{30\pi} + \frac{\ln(4/3)}{20\pi} + \frac{1}{10\pi}\right]$$

$$\therefore\ t_1 - t_4 = 740^{o}R$$

Therefore, the heater wire must be at a temperature of 3440°R, or about 1900°K. From Fig. 2 the resistivity of tungsten-rhenium wire is $75 \mu\Omega$-cm at this temperature. Then the power output of the wire is given by:

$$P = I^2 \left(\frac{\rho L}{A}\right) \tag{8}$$

We can choose a convenient size and length for the wire, for instance 10 mil diameter and 100 cm. long, and find the corresponding current and voltage required.

$$I^2 = \frac{P}{\left(\frac{\rho L}{A}\right)} = \frac{665}{75 \times 10^{-6}} \frac{\pi}{4} \frac{(.01)^2 (2.54)^2}{100} = 44.9$$

$$I = 6.7 \text{ amps}$$

$$V = \frac{665}{6.7} = 99.2 \text{ volts}$$

Using Table II we can find the life of the heater (time to evaporate 10% of the wire)

$$\text{Life} = 1630 \times 1.45 \times 10^5 \text{ hr} = 2.36 \times 10^8 \text{ hr. or } 2.7 \times 10^4 \text{ years}$$

This is actually the life of a pure tungsten heater filament, but it demonstrates that the actual wire used would be able to last considerably longer than the duration of the mission. It should also be noted that this lifetime calculation is based on full-power requirements. The thermal storage resistance jet ordinarily operates much of the time in a low power condition, so that the actual life should be much longer than calculated here.

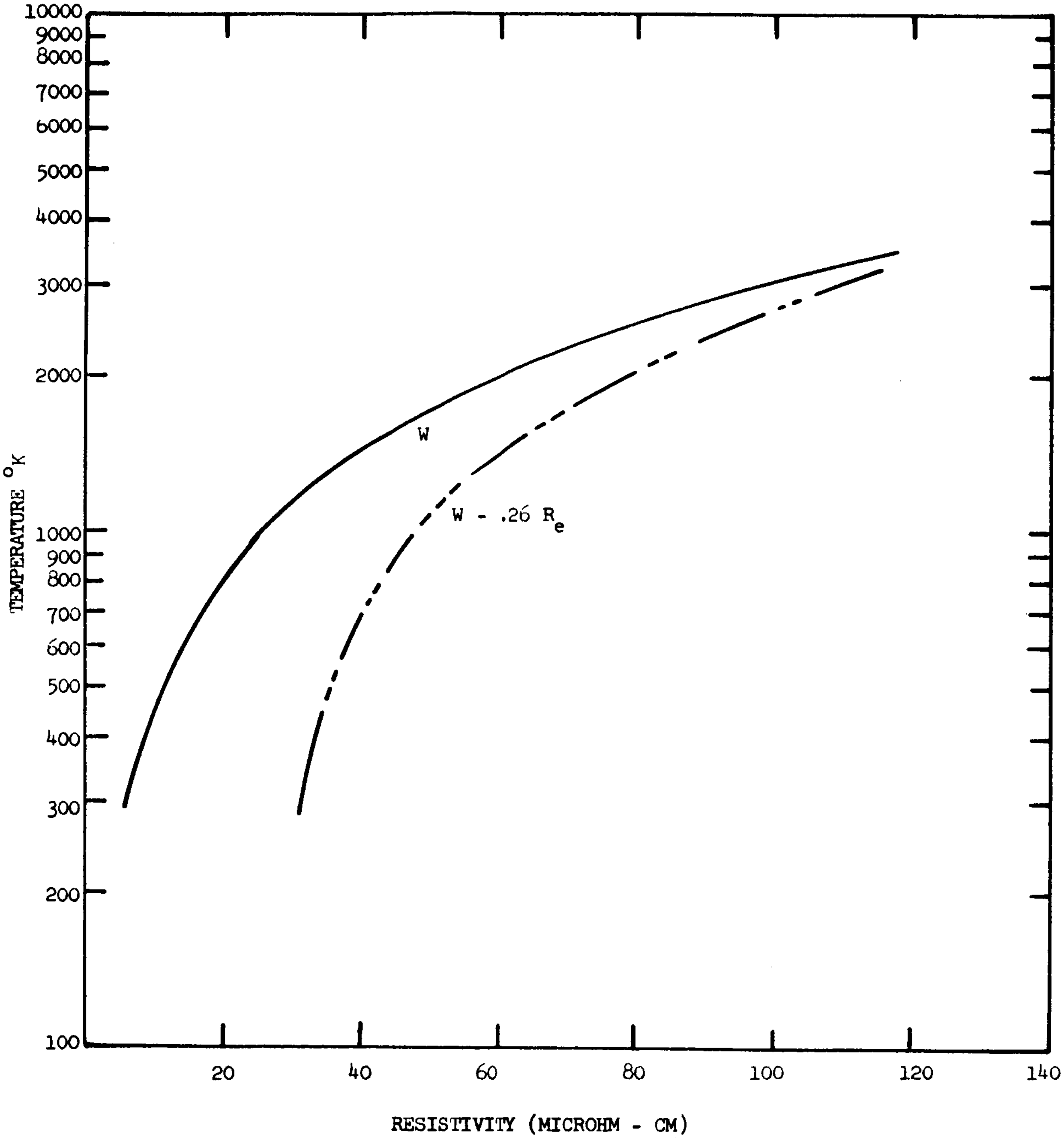


Fig. 2: Resistivity of Tungsten and Tungsten - Rhenium

TABLE II

LIFE OF A PURE TUNGSTEN FILAMENT

Wire Diameter (Inches)	Temperature (^{o}K)	Life* (Hours)
0.0006	2500	9.78
0.001	2500	163
0.002	2500	328
0.003	2500	489
0.004	2500	651
0.005	2500	815
0.006	2500	978
0.007	2500	1141
0.008	2500	1304
0.009	2500	1468
0.010	2500	1630
0.012	2500	1955
0.014	2500	2281
0.015	2500	2442
0.018	2500	2935
0.020	2500	3260
0.025	2500	4070
0.030	2500	4890
0.035	2500	5700
0.040	2500	6510
0.050	2500	8150

Below are listed factors to multiply the life at 2500^{o}K, for operation at other temperatures.

*Life is here defined as the time to evaporate 10 percent by weight.

Temperature, $^{\circ}K$	Life Factor*
1800	2.32×10^{6}
1900	1.45×10^{5}
2000	1.17×10^{4}
2100	1.23×10^{3}
2200	1.64×10^{2}
2300	25.6
2400	4.82
2500	1.00
2600	0.244

E. Heat Shielding Design

When the physical design of a resistance jet thruster has been determined, it becomes necessary to determine the amount of heat shielding required. The purpose of the heat shielding is to reduce the heat loss by radiation from the hot engine, and thereby to lower the power supply requirement.

The radiation from a hot cylindrical body (the engine) to a cylindrical enclosure (the first layer of shielding) is given by (Reference 3),

$$q_{12} = \sigma A_1 \left(\frac{1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)} \right) (T_1^4 - T_2^4) \qquad (9)$$

where q_{12} = heat radiated from body 1 to body 2 (watts)

A = surface area (cm^2)

ϵ = total emissivity (assumed constant at 0.2)

σ = Stefan - Boltzmann Constant ($5.668 \times 10^{-12} \frac{watts}{^{\circ}R^4}$)

T = Surface temperature ($^{\circ}K$)

There is also a small amount of heat conducted between the layers by the supporting wires, but this is assumed to be negligible compared to the heat radiated, for an idealized design.

The heat radiated from the first layer of heat shielding to the next layer is given by:

$$q_{23} = \sigma A_2 \left(\frac{1}{\frac{1}{\epsilon_2} + \frac{A_2}{A_3} \left(\frac{1}{\epsilon_3} - 1 \right)} \right) \left(T_2^4 - T_3^4 \right) \tag{10}$$

Similarly from the second layer to the third,

$$q_{34} = \sigma A_3 \left(\frac{1}{\frac{1}{\epsilon_3} + \frac{A_3}{A_4} \left(\frac{1}{\epsilon_4} - 1 \right)} \right) \left(T_3^4 - T_4^4 \right) \tag{11}$$

and so on, until the last (or nth) shield radiates to cold space

$$q_n = \sigma A_n \epsilon T_n^4 \tag{12}$$

Since the layers of shielding are very close together we can simplify the solution by assuming that they all have the same surface area. If we further assume that all the emissivities are the same we can add equations (9), (10), (11), ..., (12) to get:

$$nq = \sigma AE \left(T_1^4 - T_n^4 \right) + \sigma A \epsilon T_n^4 \tag{13}$$

where $nq = q_{12} + q_{23} + q_{34} + \bullet \bullet \bullet + q_n$

$$\text{and } E = \frac{1}{\frac{1}{\epsilon} + \frac{A}{A}\left(\frac{1}{\epsilon} - 1\right)} = \frac{\epsilon}{2 - \epsilon} \tag{14}$$

Rearranging gives:

$$nq = \sigma A \left(\frac{\epsilon}{2 - \epsilon}\right) \left[T_1^{\,4} + (1 - \epsilon)\, T_n^{\,4}\right] \tag{15}$$

In practice $T_1^{\,4} >> T_n^{\,4}$ so we can write

$$q = \frac{\sigma A}{n}\; T_1^{\,4} \left(\frac{\epsilon}{2\,\epsilon}\right) \tag{16}$$

This equation can be plotted for a known size resistance jet at various temperatures. In Figure 3 are shown several such curves for tantalum. Using equation (16) the amount of heat shielding necessary to maintain any desired low level of heat loss can be found for the particular engine design under consideration.

F. Engine Cool-Down Time

A measure of the effectiveness of the thermal storage resistance jet is the rate at which energy is transferred from the engine to the propellant. Therefore it is important to be able to find the amount of time that the engine remains hot after the propellant is turned on. This problem has been solved for the following range of parameters using a PANACEA computer program:

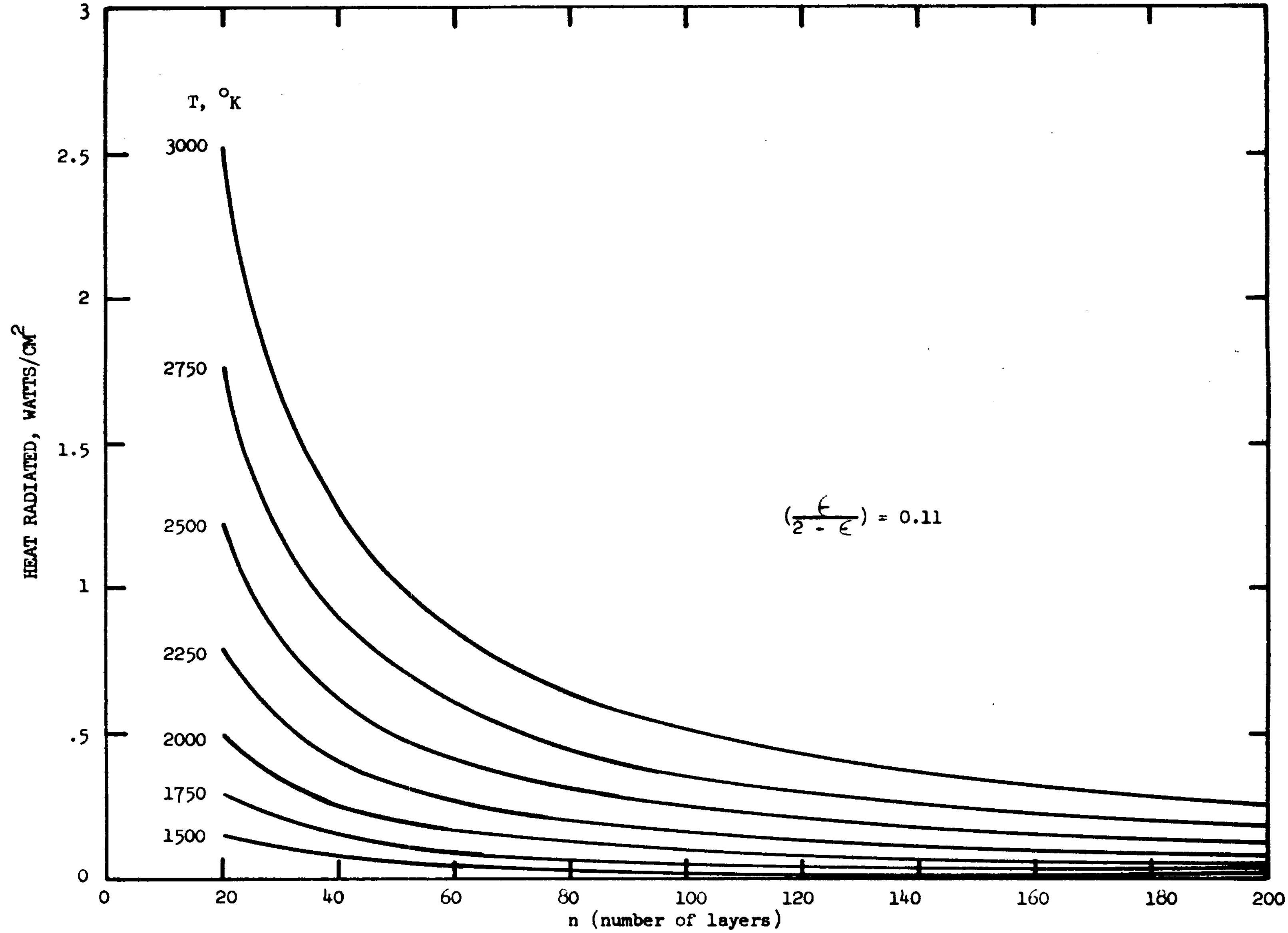


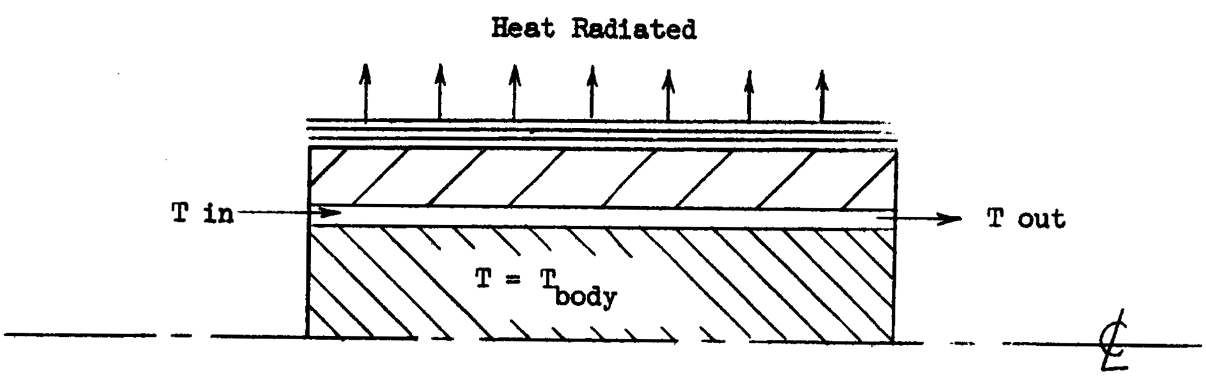
Fig. 3: Radiation Losses for Tantalum Heat Shielding

Steady-state temperature: 1500°R to 2500°R

Propellant flow heat capacity: 0.002 to 2.0 watt/°R

Engine body heat capacity: 100 watt - sec/°R

The model that was used for these calculations is shown below.



Under steady-state conditions with no flow, the electrical power supplied is equal to the rate of heat loss by radiation:

$$I^2R = \epsilon_{eff}\,\sigma\,AT_o^{\,4} \tag{17}$$

where

I^2R = electrical power, watts

ϵ_{eff} = effective emissivity of engine

σ = Stefan - Boltzmann constant, 5.4×10^{-13} watts/(cm^2 - $^oR^4$)

A = engine surface area, cm^2

T_o = steady -state body temperature, °R

Here, an effective value of the emissivity has been used based on experimental data, so as to relate the radiation heat loss to the body temperature rather than to the outside heat shield. That this can be done is shown by the heat shielding analysis in Section I.E. which shows that the radiated heat is directly proportional, for a given number of heat shields, to the body temperature.

During a propellant pulse, the energy balance becomes:

Electrical energy supplied + Energy from thruster body = Energy absorbed by propellant + Radiation energy loss

which can then be written

$$\epsilon_{eff}\,\sigma A T_o^{\,4} - W_T C_T \frac{dT}{dt} = \epsilon_{eff}\,\sigma A T^4 + \dot{W}g\, C g\,(T_{out} - T_{in}) \tag{18}$$

where W_T = weight of thruster, lb

C_T = specific heat of thruster, $\frac{\text{watt-sec}}{\text{lb}\ ^{\circ}\text{R}}$

T = temperature of thruster body, $^{\circ}$R

t = time, sec

$\dot{W}g$ = flow rate of propellant, lb/sec

C_g = specific heat of propellant, $\frac{\text{watts-sec}}{\text{lb}\ ^{\circ}\text{R}}$

T_{out} = exit propellant temperature

T_{in} = inlet propellant temperature

Because of the high heat transfer to the propellant, it can be taken that $T_{out} = T_{body} = T$. Also it should be observed that the radiation heat loss has always been referred to the body temperature, rather than the heat shield temperature, by using an experimentally determined value for an effective emissivity.

This equation can be written in finite difference form and solved for the time increment to decrease the engine temperature to any value lower than the steady state temperature:

$$\Delta t = \frac{W_T C_T \Delta T}{\epsilon_{eff}\, \sigma (T_o^4 - T^4) - \dot{W}_g C_g (T - T_g)} \tag{19}$$

This equation was solved on the computer for the range of values mentioned before. The curves in Figures 4 and 5 present these results, which then are generally applicable to various propellants and flow rates. The effect of varying thruster size and specific heat may be determined by observing that the time is a linear function of thruster heat capacity.

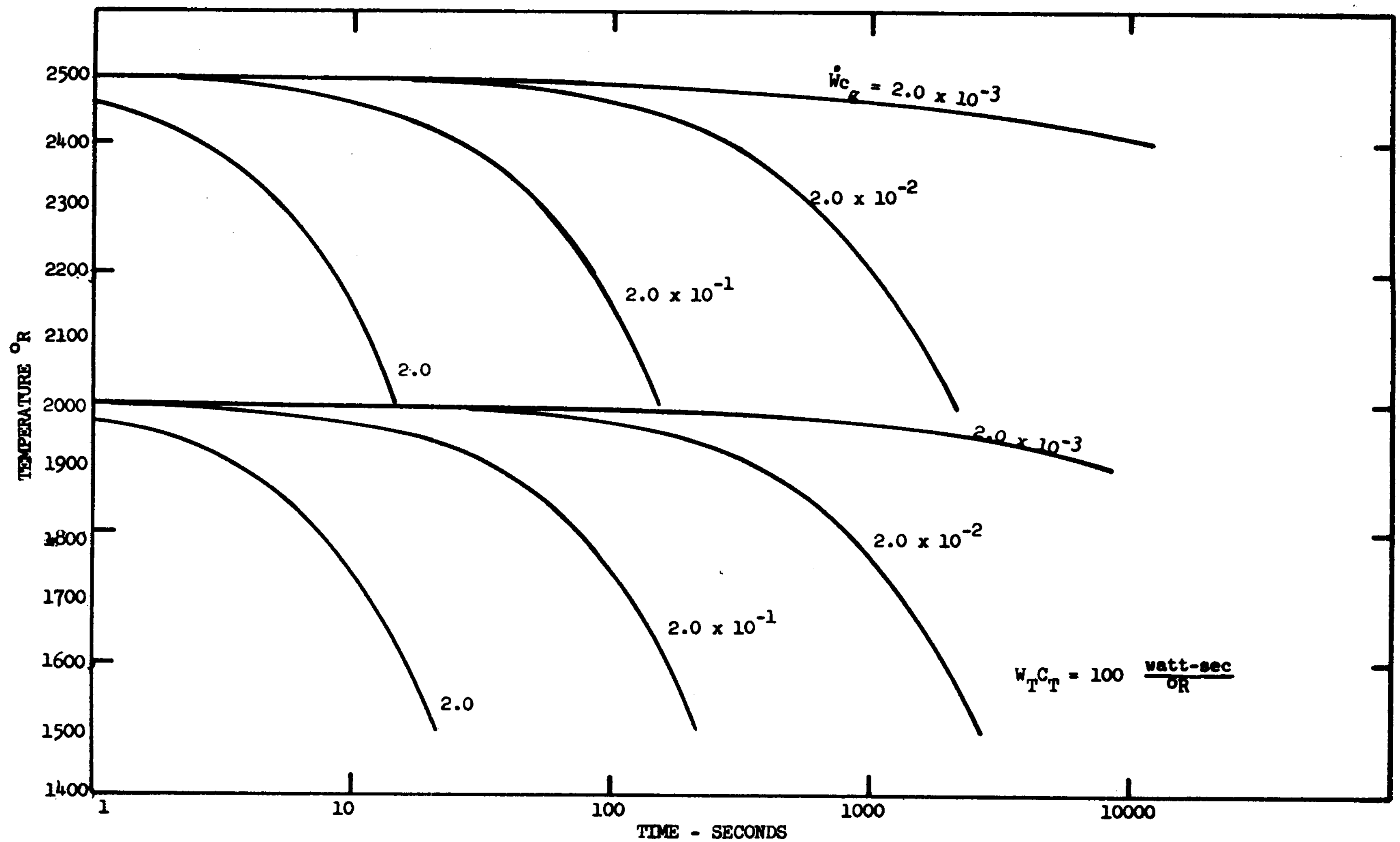


Fig. 4: Temperature Decay vs. Pulse Duration

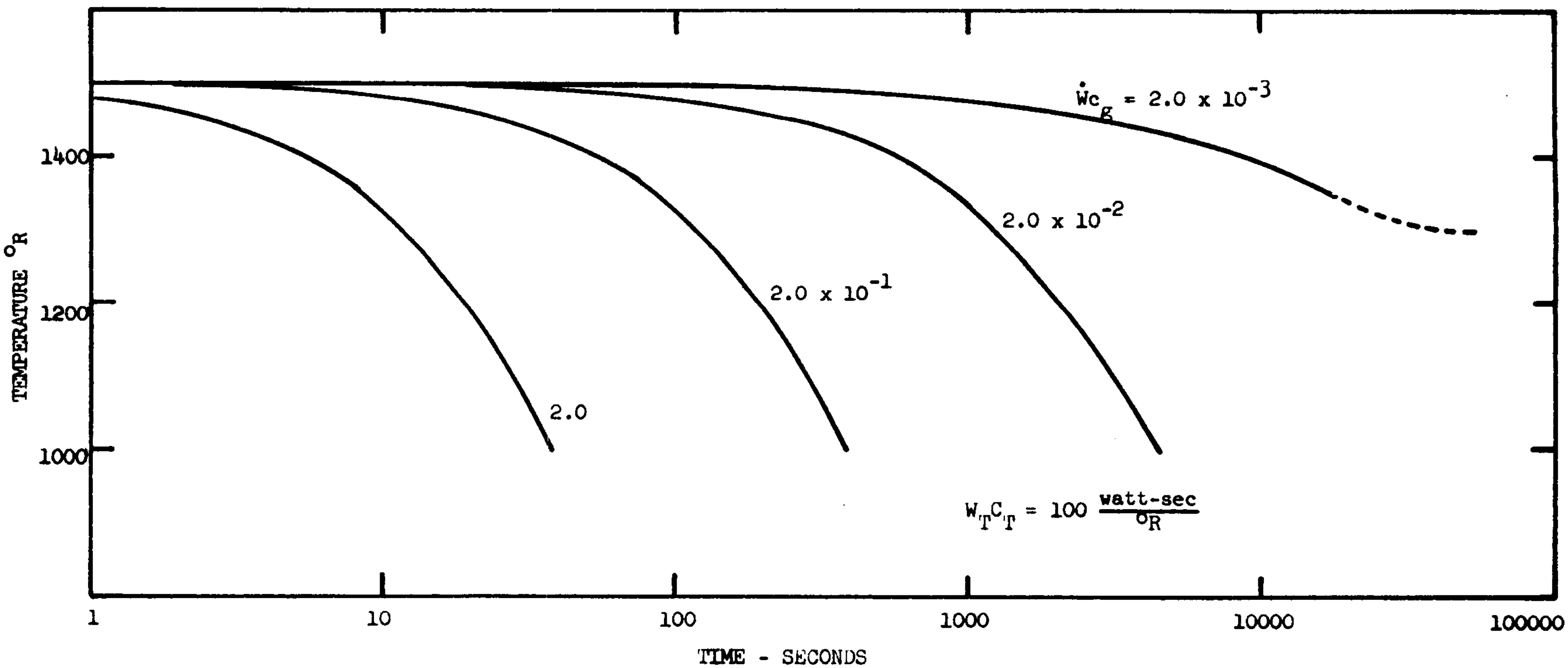


Fig. 5: Temperature Decay vs. Pulse Duration

III. TRANSIENT PERFORMANCE OF A RESISTANCE JET THRUSTER

When a tentative design for a resistance jet thruster has been developed, it is usually necessary to determine the transient characteristics of the engine. This analysis has previously been presented in Reference 4, but it is summarized in this section, and a new section for subsonic inlets is presented.

The basic approach to the evaluation of the transients has been to determine the time rate of change of thruster chamber pressure. This quantity is linearly related to the thrust and its time integral is impulse.

The model used for analysis is shown in Fig. 6. The propellant reservoir is considered to be of infinite volume and it contains propellant at constant temperature and pressure. Heat is supplied downstream of the sonic metering orifice A_1*, such that the chamber conditions are uniform throughout. The propellant escapes through the sonic throat A_3* to an expansion nozzle.

The assumptions to be used in this analysis are as follows:

1. The flow is reversible from the propellant storage to the chamber.
2. The propellant velocity in the thruster chamber is small.
3. The propellant is an ideal gas with constant specific heats.
4. The initial pressure in the thruster chamber is zero.
5. The propellant flow through the metering orifice (A_1*) is choked.
6. The final expansion is into absolute vacuum.

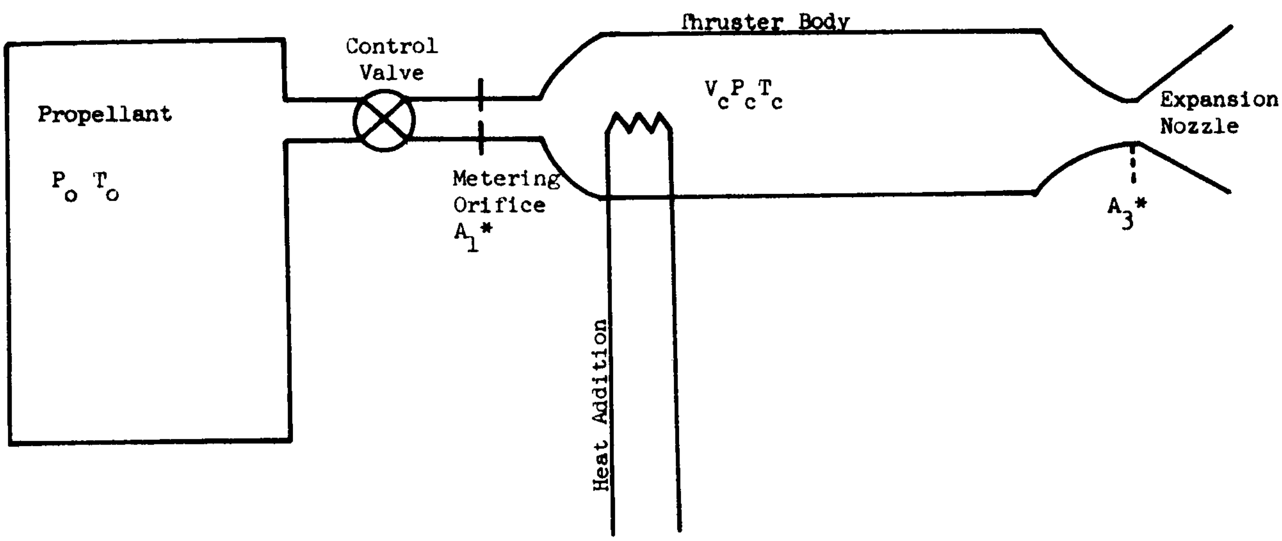


Fig. 6: Model of Thruster System

7. The stagnation temperature of the propellant in the thruster chamber is some multiple (ξ) of its temperature in the storage state.

8. The expansion process through the supersonic nozzle is isentropic.

The analysis will use the equations for conservation of mass, the perfect gas equation of state, and the sonic flow rate equation, to determine the transient pressure variation.

A. Start-Up Process

We start with an expression of assumption 7:

$$T_c = \xi \, T_o \tag{20}$$

where ξ is any number greater than unity. The instantaneous weight of gas in the chamber is:

$$W_c = g \rho c \cdot V_c \tag{21}$$

From the equation of state of an ideal gas

$$\rho c = \frac{P_c}{gRT_c} \tag{22}$$

and substituting equation (22) into equation (21):

$$W_c = \frac{P_c\ V_c}{RT_c} \tag{23}$$

Then the time variation of weight in the chamber is

$$\frac{dWc}{dt} = \frac{V_c}{R}\ \frac{d}{dt}\ (\frac{P_c}{T_c}) \tag{24}$$

Now the conservation of mass equation requires that the rate of increase of weight in the chamber is equal to the difference in the inlet and outlet weight flow rates.

$$\frac{dW_c}{dt} = \dot{W}_1 - \dot{W}_3 \tag{25}$$

The flow rate through a choked nozzle or orifice is given by:

$$\dot{W} = \frac{PA^*}{\sqrt{T_o}}\ \sqrt{\frac{kg}{R}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}} \tag{26}$$

Substituting equations (20), (24), and (26) into (25) and with the assumption that both the metering orifice (A_1*) and the nozzle (A_3*) are choked, we get:

$$\sigma\ \sqrt{\frac{kg}{R}}\left[\frac{P_oA_1^*}{\sqrt{T_o}} - \frac{P_cA_3^*}{\sqrt{\xi T_o}}\right] = \frac{V_c}{R\xi\sqrt{T_o}}\ \ \frac{dPc}{dt} \tag{27}$$

where

$$\sigma = \sqrt{(\frac{2}{k+1})^{\frac{k+1}{k-1}}} \tag{28}$$

Separating variables in equation (27) gives the chamber pressure variation as:

$$\frac{d\left(\frac{Pc}{Po}\right)}{\alpha - \left(\frac{Pc}{Po}\right)\frac{1}{\sqrt{\xi}}} = \frac{A_3^* \sigma C_o \xi}{V_c} dt \tag{29}$$

where $\alpha = \frac{A_1^*}{A_3^*}$ (30)

and the sonic velocity is given by:

$$C_o = \sqrt{gk\ RT_o} \tag{31}$$

Integrating equation (29) and using the initial condition of $P_c = 0$ at $t = 0$, the pressure variation can be expressed as:

$$\frac{Pc}{\alpha Po} = \sqrt{\xi}\ \left(1 - e^{-t/\tau}\right) \tag{32}$$

where the time constant is defined as:

$$\tau = \frac{V_c}{\sigma C_o A_3^*} \sqrt{\frac{To}{Tc}} \tag{33}$$

It can be seen that the time constant is therefore a function only of the geometry of the engine and the composition and temperature of the propellant. The steady-state pressure is given by ($\alpha Po \sqrt{\xi}$), as can be seen from Figure 7.

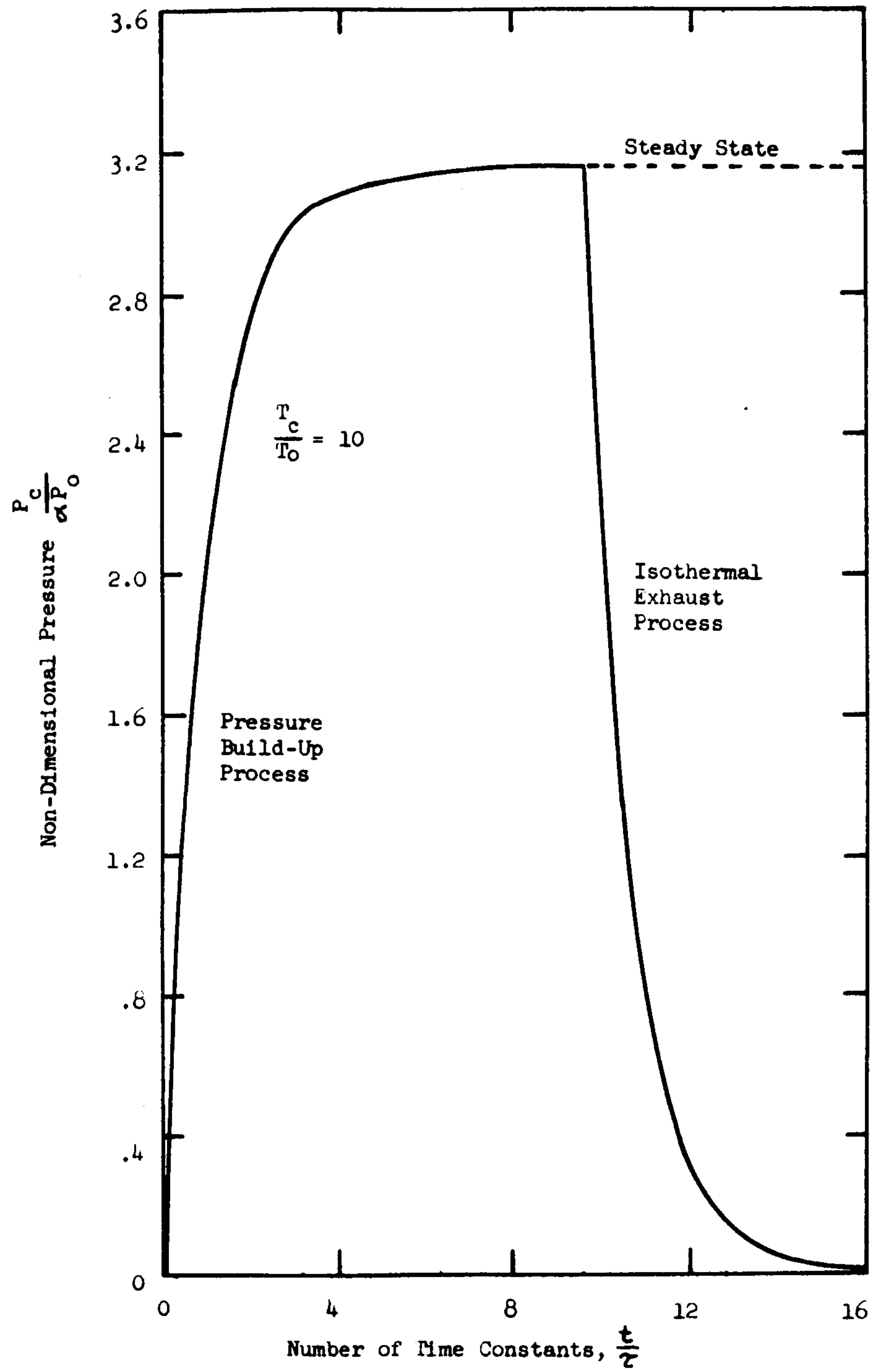


Fig. 7: Chamber Pressure Variation During Start-Up and Shut-Down Processes for Resistance Jet

B. Shut-Down Process

The thermal storage resistance jet operates with the power on at all times. When the inlet valve is closed the pressure begins to decrease until it reaches the ambient condition. Since the power is on, the expanding gas in the chamber is continuously receiving heat from the resistance heater, and the shut-down process can be approximated by an isothermal expansion process.

For an isothermal expansion process beginning at $t = \theta_1$, the chamber stagnation temperature is T_{θ_1} and remains constant. The conservation of mass requirement shown in equation (27) takes the form:

$$- \frac{V_c}{RT_{\theta_1}} \frac{dP_c}{dt} = \frac{\sigma A_3^* P_c}{\sqrt{T_{\theta_1}}} \sqrt{\frac{kg}{R}} \tag{34}$$

Separating the variables and integrating gives the expression for pressure decay:

$$\frac{P_c}{\alpha P_o} = \frac{P_{\theta_1}}{\alpha P_o} e^{-\left(\frac{t}{\tau} - \frac{\theta_1}{\tau}\right)} \tag{35}$$

A typical graph of the transient pressure variation for the resistance jet thruster is shown in Figure 7 for a chamber temperature of ten times the gas storage temperature.

C. Expressions for Thrust and Impulse

From the expressions for chamber pressure already developed, the transient variations of thrust and impulse can be readily obtained. The equation for thrust resulting from isentropic flow through a nozzle is given by:

$$F = P_c \sigma A_3^* \quad k \sqrt{\frac{2}{k-1} \left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{k-1}{k}} \right]} + A_e (P_e - P_a) \tag{36}$$

From assumption (6), $P_a = 0$. By taking a typical value for area ratio (A_e/A_3^*), the corresponding pressure ratio can be obtained from the standard isentropic flow tables.

The non-dimensional instantaneous thrust during the transient state can be expressed by:

$$\frac{F}{P_c A_3^*} = \sigma k \sqrt{\frac{2}{k-1} \left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{k-1}{k}} \right]} + \frac{A_e}{A_3^*} \cdot \frac{P_e}{P_c} \tag{37}$$

The right-hand side of this equation is then constant for a particular propellant species and a specified area ratio (A_e/A_3^*). This constant can be defined as K_1 so that:

$$F = K_1 \; P_c A_3^* \tag{38}$$

Substituting from equation (32) gives the thrust variation during the start-up process:

$$F = \alpha \; P_o A_3^* \; K_1 \sqrt{\xi} \; (1 - e^{-t/\tau}) \tag{39}$$

which shows that the thrust is directly proportional to the chamber pressure. The same is true for the pressure decay processes, where the thrust is given by:

$$F = K_i A_3^* P_{\theta_1} e^{-\left(\frac{t}{\tau} - \frac{\theta_1}{\tau}\right)} \tag{40}$$

Since the impulse is simply the integration of thrust over a specified time period, the calculation of impulse can now be accomplished. Thus, for the start-up process the impulse is given by:

$$I = K_1 \cdot \sqrt{\xi} \left(\frac{t}{\tau} + e^{-t/\tau} - 1\right) \tag{41}$$

where

$$K_1 = \frac{K_i \alpha P_o V_c}{\sigma C_o} \sqrt{\frac{T_o}{T_c}} \tag{42}$$

For isothermal expansion, the impulse is given by:

$$I = \frac{K_1 P_{\theta_1}}{\alpha P_o} \left[1 - e^{-\left(\frac{t}{\tau} - \frac{\theta_1}{\tau}\right)}\right] \tag{43}$$

A graph of these expressions for impulse is shown in Fig. 8. Note that the contribution to the impulse from the expansion of residual gas in the shut-down is relatively small for both kinds of expansion processes.

D. Application of the Transient Analysis

One of the more important characteristics of a resistance jet for the controls engineer is the time constant. The analytical treatment has established that the time constant for the resistance jet has the form:

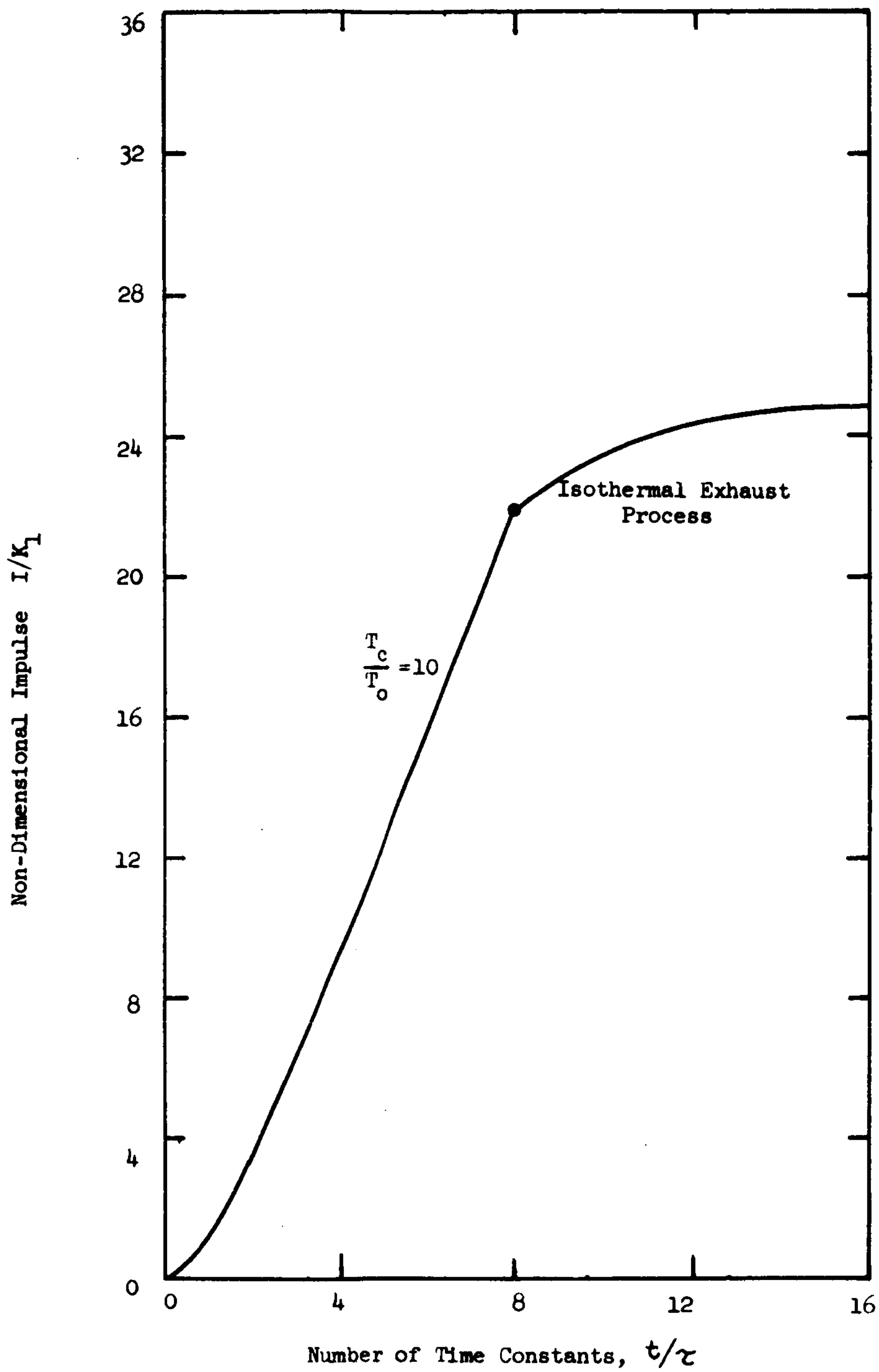


Fig. 8: Delivered Impulse for Resistance Jet Cycle

$$\tau = \frac{V_c}{\sigma C_o A_3^*} \sqrt{\frac{T_o}{T_c}}$$

It is instructive to examine the effect of the important variables on τ. The chamber volume, V_c, should be small if a short time constant is desired. Volumes as low as 0.1 cm^3 are attainable in resistance jet designs. The value of σ is dependent on the propellant and varies from 0.58 for k = 1.4 to 0.54 for k = 1.67.

Increase of the nozzle throat area, A_3^* will decrease τ. However, this approach will tend to increase chamber volume and also to increase the area from which heat is radiated. This latter effect is detrimental to thruster efficiency. Further, increasing A_3^* causes a decrease in the steady-state value of chamber pressure, P_c. This in turn results in increased fluid dynamic losses in the exhaust nozzle with concurrent losses in performance.

The stagnation sonic velocity, $C_o = \sqrt{kgRT_o}$, should be large for τ to be small. The effect of T_o of course vanishes, although from a practical point of view the power required to heat the gas to T_c depends on the temperature interval $(T_c - T_o)$. The choice of propellant will establish the value of the specific heat ratio, k, which in any event does not strongly influence the value of C_o.

The major effect on the time constant for a particular engine is to be found in the specific gas constant R, which is inversely proportional to the propellant molecular weight. The time constant is therefore directly proportional to the square root of propellant molecular weight.

Representative values of the several terms in the expression for T are:

V_c	of order	$1\ cm^3$
σ	of order	0.5
C_o	of order	10^5 cm/sec (mol. wt. = 2)
$\sqrt{T_o/T_c}$	of order	1/3
$A_3{}^*$	of order	$.01\ cm^2$

These lead to a value of the time constant $\tau = 7 \times 10^{-4}$ seconds.

The time constant for a cold gas system, typically with nitrogen propellant (mol. wt. = 28), would be larger by the factor $\sqrt{T_c/T_o}$, as well as by the factor $\sqrt{28/2}$, for a net order of magnitude increase to approximately 7×10^{-3} seconds. Therefore, a resistance jet system will attain steady-state about ten times as fast as a cold gas system.

We can now proceed to a discussion of the several rate equations which show the transient behavior. In what follows, chamber pressure will be the variable under consideration. Because of the linear relation between chamber pressure and thrust, comments on the former will be applicable to the latter. It should be pointed out that this relation is significant in considering methods for experimental verification. Thrust measurement at the low thrust levels that are required for attitude control missions, is difficult. The measurement of thrust transients represents a problem of even greater magnitude. On the other hand, there are available instruments for the reliable measurement of low pressures and associated transients.

In general terms the pressure transient are expressed as:

$$\frac{P_c}{\alpha P_o} = \text{constant + exponential function of time.}$$

The constant represents the steady-state value of the dimensionless chamber pressure and the exponential represents the rate at which the steady state is achieved. For start-up the constant has the value $\sqrt{T_c/T_o}$ and it is zero for shut-down. The exponential has the form $e^{-t/\tau}$ both for start-up and for shut-down.

There are a number of interesting results that can be obtained from examination of Fig. 7 and a consideration of the physical processes. The area under the curve is proportional to the impulse delivered during this time. The maximum impulse that could be delivered is the rectangular area represented by the product of the steady-state pressure and the time. The ratio of delivered to maximum impulse is 0.8 for this graph. Further, because of the shape of the shut-down process curve the impulse delivered during the shut down process following propellant valve closure is 0.2 times the maximum impulse that could be delivered during this same time period. The total impulse delivered over a cycle in which steady state is achieved is therefore the product of the steady-state thrust (which is proportional to chamber pressure) and the propellant valve on-time. If steady-state is not achieved; that is, if the valve is closed before five time constants have elapsed, the total impulse delivered is again the product of the steady-state thrust and the propellant valve on-time. This may be shown as follows:

The total impulse for a cycle is given by:

$$I_{total} = \int_0^{\theta_1} C_1 \left(1 - e^{-t/\tau}\right) dt + \int_{\theta_1}^{\infty} C_2 e^{-(t/\tau - \theta_1/\tau)} dt \qquad (44)$$

where $C_1 = K_i \alpha P_o A_3^*$

$$C_2 = K_i A_3^* P_{\theta_1} = K_i A_3^* P_o \alpha (1 - e^{-\theta_1/\tau})$$

This integrates to:

$$I_{total} = K_i \alpha P_o A_3^* \theta_1 \sqrt{\xi}$$

but $F_{steady\text{-}state} = F_\infty = K_i \alpha P_o A_3^* \sqrt{\xi}$

$$\text{so} \quad I_{total} = F_\infty \cdot \theta_1 \qquad (45)$$

Consider now that a thruster is required to function in undisturbed limit cycle operation as well as in maneuver and acquisition. The maneuver requirement will establish the minimum thrust level. If the propellant valve has a fixed minimum cycle time then the minimum impulse bit that can be delivered will be a function of the time constant. Thus, if F_{θ_1} is the maximum thrust developed during the minimum valve cycle time t_V, then the minimum impulse bit that can be delivered is $I_{MIN} = F_\infty \cdot t_V$, where $F_\infty = \dfrac{F_{\theta_1}}{1-e^{-t_V/\tau}}$.

In undisturbed limit cycle operation it is desirable to keep I_{MIN} as small as possible. This results in increased time within the deadband and a commensurate reduction in the number of cycles and in the propellant consumption.

To achieve a small value of minimum impulse bit, it is necessary to have a fast-acting valve and not too high a value of steady-state thrust. From the control engineer's viewpoint, a small time constant is desired. The advantage of a short time constant is that it means the system is fast-acting, that is it comes to a high thrust level rapidly. Therefore, it is able to correct errors quickly and keep the vehicle attitude within prescribed limits for a high percentage of the mission time with less propellant expended.

In addition to the above effect, there is an effect of the time constant on the boundary layer losses of the engine. As the chamber pressure increases towards steady-state, the thickness of the boundary layer decreases and therefore the thrust increases faster than the pressure. This effect is shown by Fig. 9 which is a graph of thrust as a function of chamber pressure for a typical engine design, obtained by using the boundary layer computer program. Therefore, as the chamber pressure increases with time, the thrust will be less than predicted by equation (39) because of the boundary layer losses. This effect can be used to advantage in obtaining a smaller value of minimum impulse bit by increasing the value of the time constant. In other words, if the calculated ideal value of the minimum impulse bit, $(F_{\infty} \cdot t_V)$, is too large for a given application, it may be lowered by increasing the time constant (such as by increasing the chamber volume). This will mean that, during a transient pulse, the engine will exert less thrust because it spends more time in a lower thrust condition.

In summary, the design goals for a resistance jet thruster are a small time constant, a short valve time, and as high a level of steady-state thrust as is commensurate with a desired minimum impulse bit. If necessary, the time constant

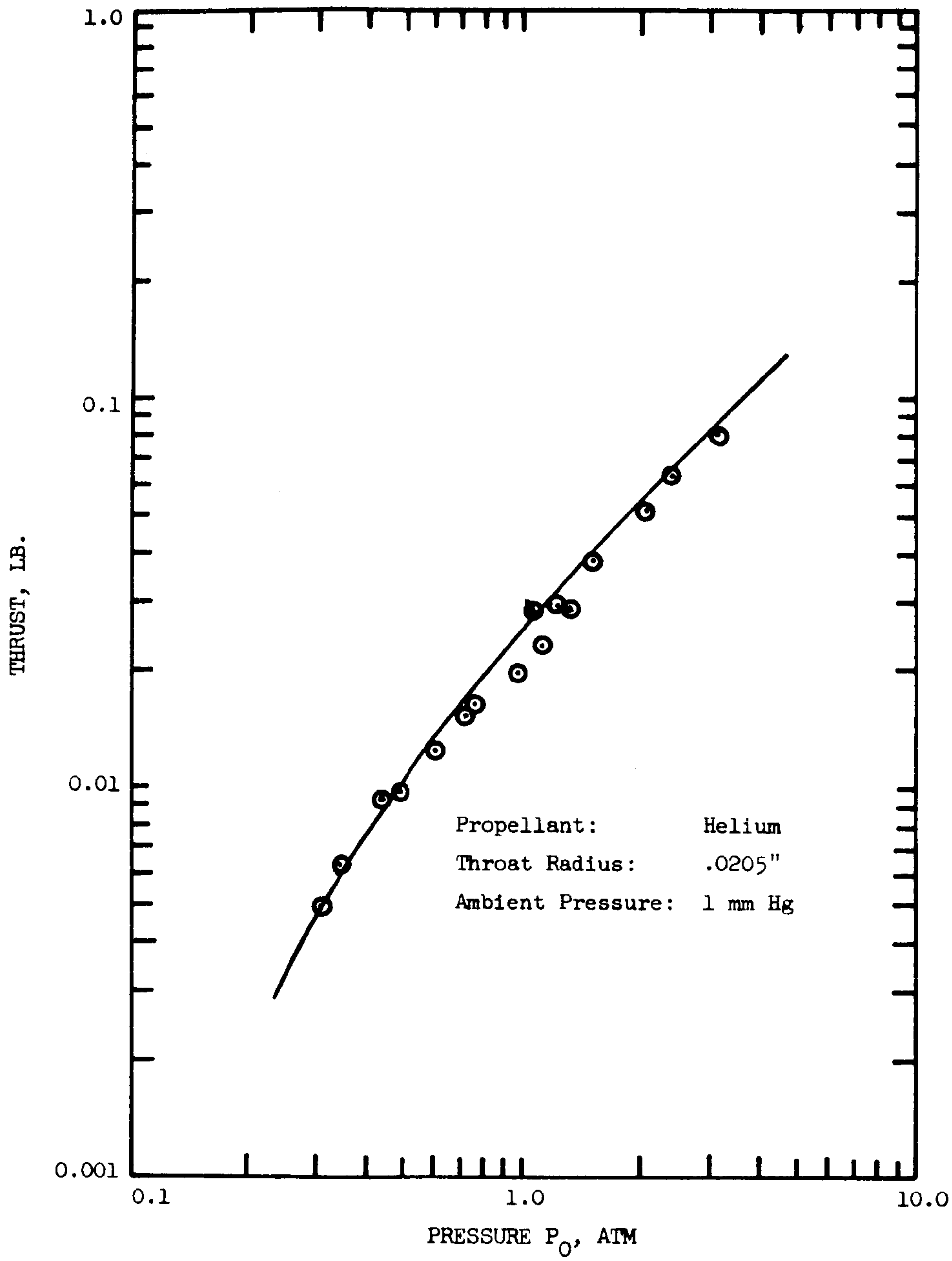


Fig. 9: Experimental Thrust Variation as a Function of Chamber Pressure

can be increased to lower the actual value of the minimum impulse bit. It should be mentioned that it is often difficult to guess the results of changes in thruster design or propellant; and the recommended procedure is to carry out the actual calculations, as given in this report, to find the effects of any design modifications.

E. Subsonic Flow Correction

If the inlet orifice of a resistance jet is not choked, then the transient analysis must be modified. Equation (25) remains the same:

$$\frac{dW_c}{dt} = \dot{W}_1 - \dot{W}_3 \tag{46}$$

but now

$$\dot{W}_1 = C_1 A_1 \sqrt{2\rho_o (P_o - P_c)} \tag{47}$$

where C_1 is an orifice coefficient (usually about 0.6). Then by substituting the flow rate expressions into equation (46) and also using the following:

$$\sigma = \sqrt{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}$$

$$\alpha = A_1/A_3^*$$

$$C_o = \sqrt{kgRT_o}$$

$$W_c = P_c V_c / RT_c$$

we have the following:

$$\frac{dW_c}{dt} = C_1 A \sqrt{2\rho \Delta P} - \frac{P_c A_3^*}{\sqrt{T_c}} \sqrt{\frac{kg}{R} \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}$$

Rearranging gives:

$$\frac{\frac{d}{dt}\left(\frac{P_c}{P_o}\right)}{K_1\sqrt{1 - P_c/P_o} - K_2 P_c/P_o} = \frac{A_3^* R \xi T_o}{V_c}$$

where

$$K_1 = C_1 \alpha \sqrt{2/RT_o} \quad \text{and} \quad K_2 = \sigma \sqrt{kg/R \xi T_o}$$

Therefore:

$$\frac{d}{dt}\left(\frac{P_c}{P_o}\right) \approx \frac{A_3^* R \xi T_o \sigma}{V_c} \sqrt{\frac{kg}{RT_o}} \left[\frac{C_1 \alpha \sqrt{2}}{\sigma \sqrt{k}} \left(1 - \frac{1}{2} \frac{P_c}{P_o}\right) - \frac{1}{\sqrt{\xi}} \frac{P_c}{P_o}\right]$$

because

$$\sqrt{1 - P_c/P_o} \approx \left(1 - \frac{1}{2} P_c/P_o\right)$$

with an error of about five percent for $P_c/P_o = 0.5$ and twenty percent for $P_c/P_o = 0.86$.

Then:

$$\frac{d}{dt}\left(\frac{P_c}{P_o}\right) \approx \frac{A_3{}^*\sigma\xi}{V_c}\sqrt{kgRT_o}\left[\frac{C_1\alpha\sqrt{2}}{\sigma\sqrt{k}}\left(1 - \frac{1}{2}\frac{P_c}{P_o} - \frac{1}{\sqrt{\xi}}\frac{P_c}{P_o}\right)\right] \tag{48}$$

and therefore, the equation becomes

$$\frac{P_c}{P_o} = \left\{\frac{C_1\alpha\sqrt{2}}{\sigma\sqrt{k}} - \left[e^{-\left(\frac{C_1\alpha\sqrt{2\xi}}{2\sigma\sqrt{k}} + 1\right)\frac{A_3{}^*\sigma\sqrt{\xi}C_o}{V_c}t}\right]\left[e^{-K_o\left(\frac{C_1\alpha\sqrt{2}}{2\sigma\sqrt{k}} + \frac{1}{\sqrt{\xi}}\right)}\right]\right\} \cdot \left(\frac{1}{\frac{C_1\alpha\sqrt{2}}{2\sigma\sqrt{k}} + \frac{1}{\sqrt{\xi}}}\right) \tag{49}$$

where K_o can be evaluated from the initial condition.

The corrected time constant therefore becomes:

$$\tau' = \frac{V_c}{\sigma C_o A_3{}^*}\sqrt{\frac{T_o}{T_c}} \cdot \left(\frac{1}{1 + \frac{C_1\alpha\sqrt{2}}{2\sigma\sqrt{k}}\sqrt{\frac{T_c}{T_o}}}\right) \tag{50}$$

where C_1 is the inlet orifice coefficient.

IV. EFFECTS OF THRUSTER ON THE VEHICLE

A. Angular Velocity Change

A series of curves has been calculated for use in determining the angular velocity change imparted to a satellite by a resistance jet thruster. These calculations were made on a family of vehicles each weighing 2000 lbs. with moments of inertia varying from 100 slug-ft^2 to 1000 slug-ft^2.

The derivation of the equation is as follows:

Work done on satellite by thruster = (moment)x(angular displacement in radius)

$= (F \times R) \times (2\pi N)$

where F = thrust of resistance jet

R = radius in ft.

N = number of revolutions

By conservation of energy, work done = increase in K.E. of rotation

$$(F \times R) \times (2\pi N) = (\tfrac{1}{2}) \; I \, (\omega_1^2 - \omega_o^2) \tag{51}$$

where ω_o = initial angular velocity

ω_1 = final angular velocity

But $2\pi N = \omega_{AV} t$ where ω_{AV} = average velocity during pulse and t = time of pulse.

So $$(F \times R) \frac{(\omega_0 + \omega_1)}{2} \quad t = \left(\frac{1}{2}\right) I (\omega_1 + \omega_0) \quad (\omega_1 - \omega_0) \tag{52}$$

$$\therefore \underline{\underline{FRT = I \, (\omega_1 + \omega_0)}}$$

Using this equation ω_1 was found under the following conditions:

$\omega_0 = 0$

F = .01 to .05 lb

R = 2.5 ft.

I = 100 to 1000 slug ft^2

t = 1 to 10 sec.

The curves for these conditions are shown in Figs. 10 to 13.

This analysis can therefore be used to find the total change of angular velocity available for a particular design of thruster operating in conjunction with a given vehicle.

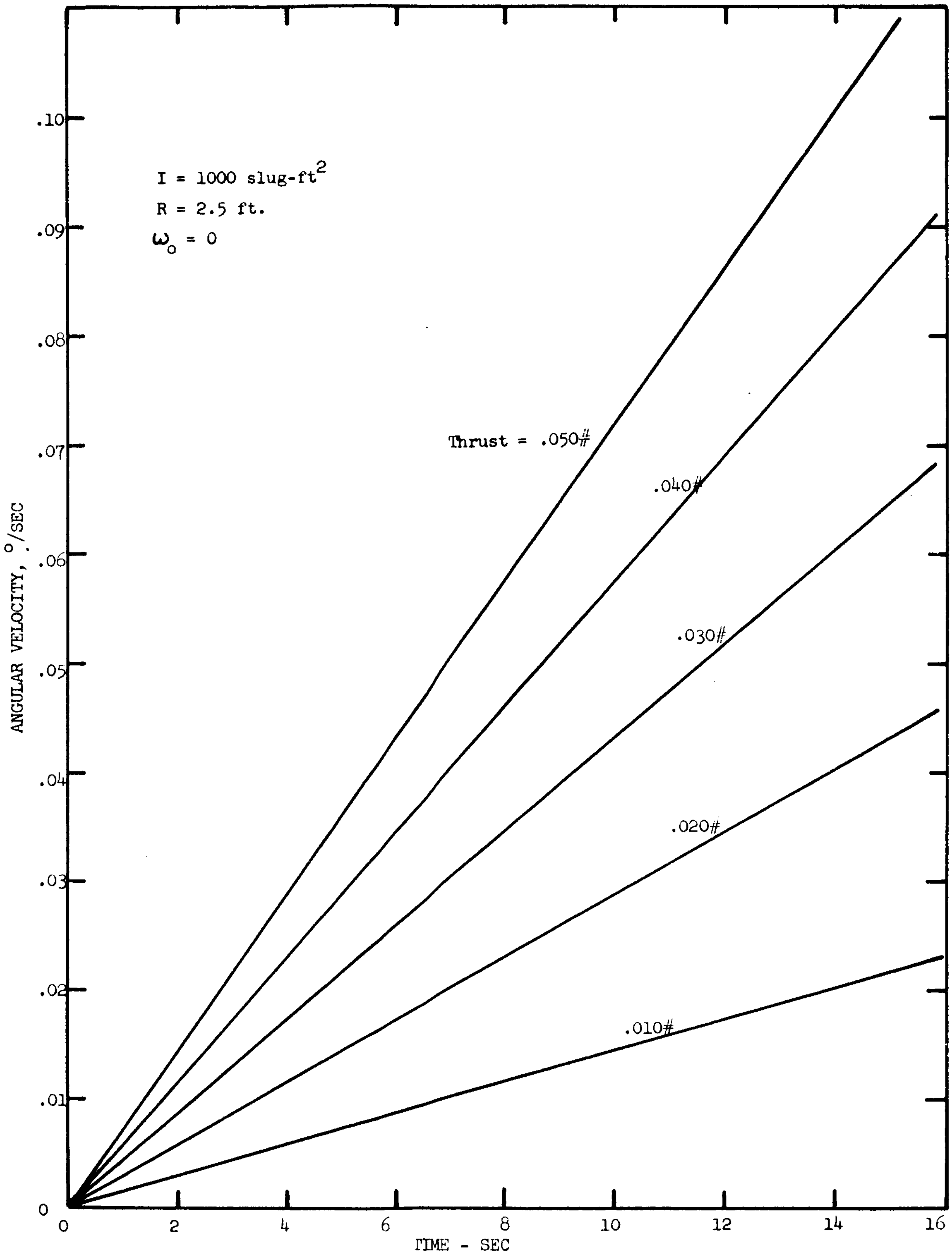


Fig. 10: Angular Velocity Imparted by Thruster

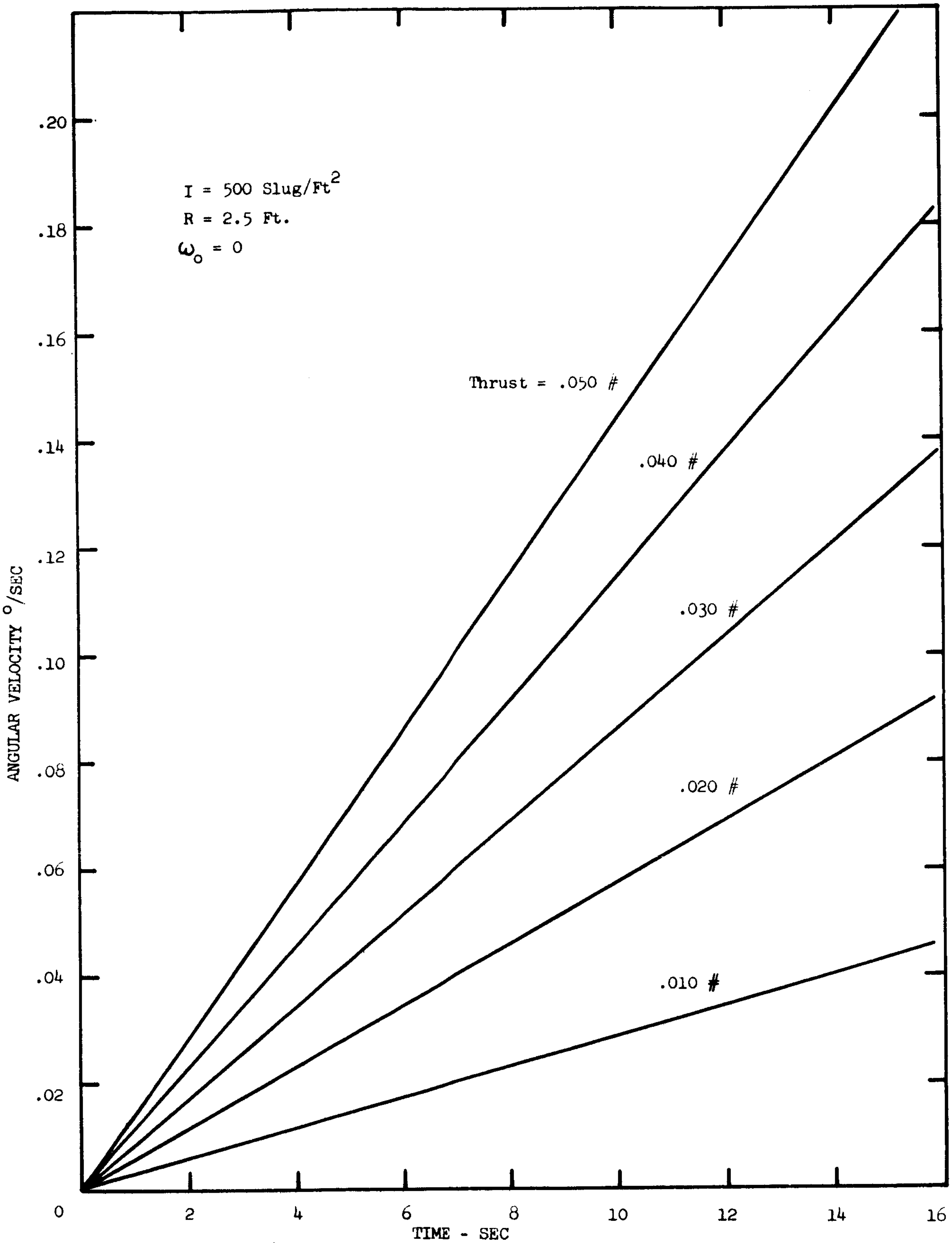


Fig. 11: Angular Velocity Imparted by Thruster

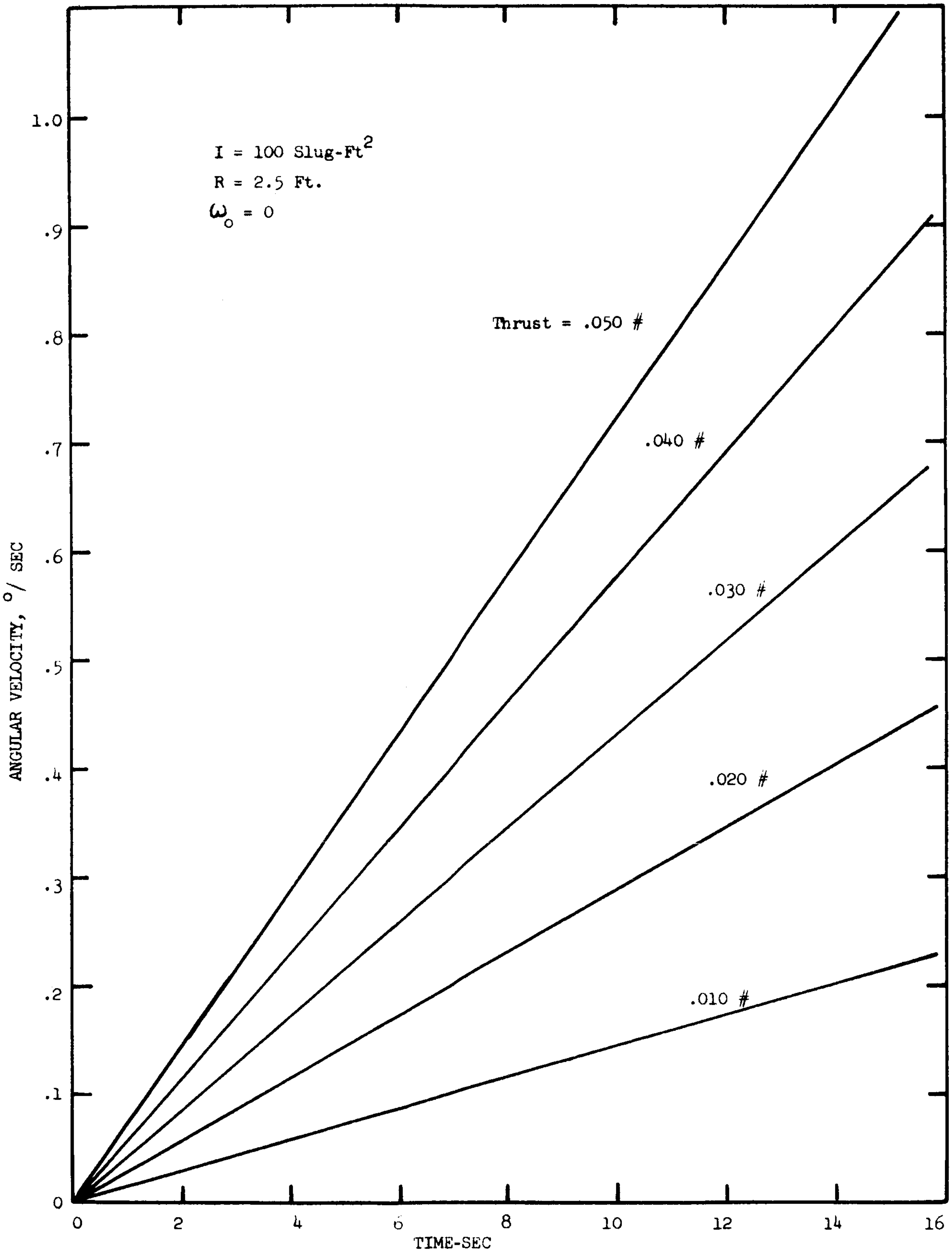


Fig. 12: Angular Velocity Imparted by Thruster

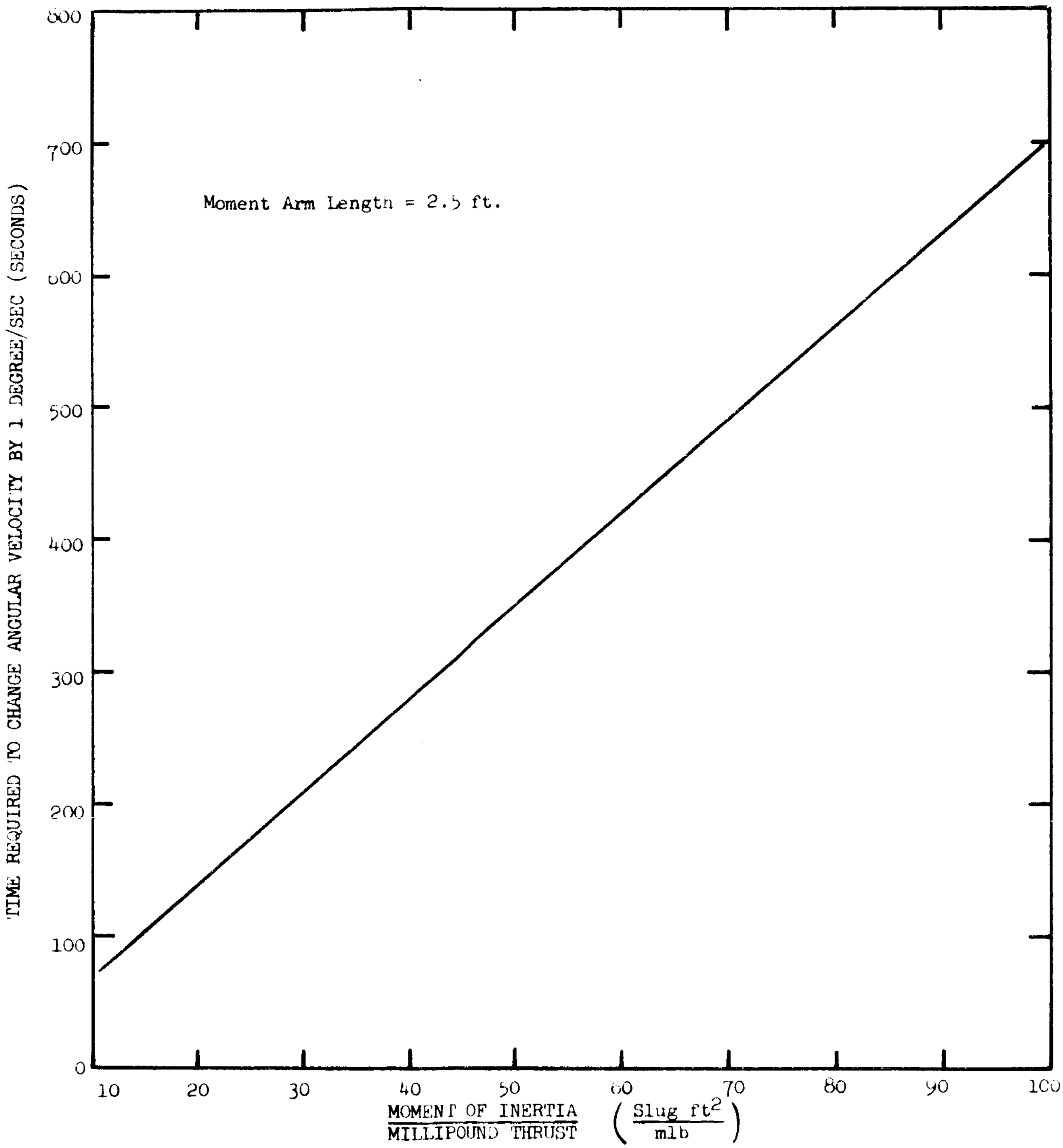


Fig. 13: Unit Angular Velocity Acceleration

B. Optimization of the Resistance Jet System for a Particular Vehicle

The previous section of this report has given a design procedure for a resistance jet attitude control system. The procedure was to assume a thrust level and then choose propellant temperature and pressure from practical considerations. Then, based on these parameters, the thermodynamic design was optimized by selecting the proper size and shape of the exhaust nozzle. However, it should be noted that no attempt was made to optimize the overall resistance jet system.

A recent analysis has been made to determine the optimum specific impulse for a resistance jet system based on the overall vehicle weight penalty (Reference 5). The use of too high a specific impulse will result in extremely high power requirements and a correspondingly high investment in power supply weight. Conversely, too low a specific impulse will result in excessively high propellant requirements. The optimum specific impulse can be obtained by minimizing the sum of the propellant, propellant tankage, and power supply weight requirements for a specified level of mission propulsion requirements.

This section contains the results of an investigation of the optimum specific impulse of thermal storage resistance jets for station keeping and attitude control missions. The analysis illustrates the dependence of the optimum specific impulse upon the major mission parameters including mission characteristic velocity, vehicle gross weight, and number of thrusters required per vehicle.

Analysis

The vehicle weight fraction chargeable to the station keeping and attitude control propulsion system will be assumed to include the system power supply, the propellant required by the mission, and the propellant storage tank weight:

$$W_s/W_o = w\ (P/W_o) + (1 + w_T)\ (W_{PP}/W_O) \tag{53}$$

The system power requirements will be dependent upon the level of steady-state thermal radiation losses which will vary with the fourth power of the engine operating temperature:

$$P = NK_1\ T^4 \tag{54}$$

The constant K_1 is a function of the engine design and the effectiveness of thermal insulation obtained. A value of $K_1 = 6.25(10)^{-13}$ watts/$^oR^4$ was assumed, corresponding to a loss of 10 watts per thruster at an operating temperature of 2000^oR. Equation (54) should be adequate for the relatively small vehicles and low characteristic velocity missions investigated in this study. For somewhat higher values, it may become necessary to add a term to reflect the jet power expended during the many propulsion pulses.

For a given temperature and power level, the specific impulse obtainable from a particular propellant can be represented empirically by an equation of the form:

$$T = A\,(1 + B\,I_{sp}^{\;2} + CI_{sp}^{\;4}) \tag{55}$$

The propellant weight fraction can be expressed in terms of the total mission characteristic velocity and the engine specific impulse:

$$W_{PP}/W_o = 1 - W_1/W_o = 1 - e^{-\Delta V/g\,I_{sp}} \tag{56}$$

Equations (54), (55), and (56) can be substituted into equation (53), the result differentiated with respect to specific impulse, and equated to zero in order to minimize the total system weight. The resulting equation can then be written in the form:

$$X = \frac{(1+w_T)(W_1/W_o)W_o\Delta V}{Nwg} = 8K_1A^4I_{sp}\,(B + 2C\,I_{sp}^{\;2})\,(1 + BI_{sp}^{\;2} + CI_{sp}^{\;4})^3 \tag{57}$$

The right-hand side of equation (57) can be evaluated over the specific impulse range of interest for any propellant and represented by an equation of the form:

$$I_{sp} = D + E\,\ln X + F\,\ln^2 X \tag{58}$$

The mass ratio can be set equal to 1.0 without introducing any significant error in the calculated value of specific impulse from equation (58). The left-hand side of equation (57) can therefore be simplified to:

$$X = \frac{(1 + w_T)\,W_o\Delta V}{Nwg} \tag{59}$$

The resulting equations (53) through (56), (58), and (59) can be used to generate the desired specific impulse requirement and the other system parameters as a function of vehicle gross weight, mission characteristic velocity, number of engines per thruster, powerplant specific weight, and propellant tankage factor.

For hydrogen propellant, the following empirical values can be used:

Propellant tankage factor, $w_T = 100\%$

Power supply weight, $w = 0.3$ lb/watt

Empirical coefficients, $A = 138.333$

$B = 5.392 \times 10^{-5}$

$C = 0$

$D = 276.694$

$E = 3.7415$

$F = 3.8639$

These empirical factors can then be used to find curves of optimum specific impulse as a function of vehicle gross weight and mission characteristic velocity. Also curves of total impulse per thruster can be generated as a function of vehicle gross weight and characteristic velocity, where the total impulse is given by:

$$I_T = W_o \, I_{sp} \, (W_{PP}/W_o) \tag{60}$$

These calculations can be carried out to demonstrate the optimum application for the example engine designed previously in this report. The right-hand side of equation (57) can be evaluated so that, for I_{sp} = 596 sec,

$$\frac{(1 + w_T)\left(\frac{W_1}{W_o}\right) W_o \Delta V}{Nwg} = 171{,}077$$

Since $w_T = 1.0$ for hydrogen and $\frac{W_1}{W_o} \approx 1.0$, we have $\frac{W_o \Delta V}{N} = 826{,}302$

Therefore, for a 1000 lb vehicle with a characteristic velocity of 826 ft./sec. for each thruster, the hydrogen resistance jet engine with a specific impulse of 596 sec. imposes the smallest overall weight penalty. This analysis has therefore defined the range of missions (i.e., product of gross weight and characteristic velocity) for which a particular engine is optimum.

An alternative approach to the problem, and one which should probably be used for flight hardware, is first to define the mission and then design the engine around the optimum specific impulse. This approach can be readily employed, when the variation of effective specific impulse is known as a function of temperature, by employing the methods given in this analysis. Therefore an optimum system can be designed for any particular vehicle application.

V. SUMMARY AND CONCLUSIONS

This report has presented analytical methods of establishing the thermodynamic and physical design of a resistance jet thruster for attitude control of a space vehicle. It also includes methods for analyzing the transient performance of the thruster and guidelines for determination of optimum performance levels. Most of these analyses have already been used for actual thruster design and their accuracy has been experimentally validated.

Some interesting conclusions can be drawn from the results of these analyses:

1) The optimum specific impulse level for a particular thruster application may be somewhat lower than expected, because a high specific impulse may cause excessive power requirements.

2) The flow path necessary for maximum heat transfer to the propellant will probably be much shorter than is required to allow ample space for the heater wire.

3) A large number of layers of heat shielding will probably be required to decrease the radiation heat loss to a sufficiently low level.

4) Design goals for good response generally include a small time constant, short valve time, and as high a thrust level as is compatible with a small minimum impulse bit.

5) It takes several minutes for an engine with propellant flowing to cool down enough to experience a 10% loss in thrust, which means that the thermal storage type of resistance jet thruster is well-suited to most applications of attitude control and station keeping.

6) The lifetime of a resistance jet engine is normally determined by the lifetime of the heater wire, which is typically many times longer than the duration of the mission.

APPENDIX A. NOZZLE DESIGN OPTIMIZATION RESULTS

NOZZLE NUMBER 99 PROPELLANT HYDROGEN

DESIGN SPECIFICATIONS

NOZZLE DATA

THROAT RADIUS	1.1100-02 IN	IDEAL EXIT RADIUS	4.9630-02 IN
IDEAL LENGTH	8.0000-01 IN	MAXIMUM DIVERGENT ANGLE	2.5000+01 DEG
CONVERGENT RADIUS	1.5540-02 IN	DIVERGENT RADIUS	4.4400-03 IN
WALL TEMPERATURE	2.5000+03 DEG R	THROAT AREA	3.8708-04 IN2

THERMODYNAMIC DATA

STAGNATION PRESSURE	5.0000+00 ATM	AMBIENT PRESSURE	1.0000+00 MM HG
STAGNATION TEMPERATURE	2.7000+03 DEG R	AMBIENT TEMPERATURE	5.4000+02 DEG R
FRACTION DISSOCIATED	4.4025-06	FRACTION IONIZED	4.4721-16
ENTHALPY	1.5694+04 B/LB	BASE ENTHALPY	7.4308+03 B/LB
FLOW PER UNIT THROAT AREA	1.9621-01 LB/SEC-IN2	THROAT TEMPERATURE	2.2940+03 DEG R
IDEAL SPECIFIC IMPULSE	6.7426+02 SEC	THERMODYNAMIC EFFICIENCY	1.1383+00
SPECIFIC GAS CONSTANT	9.8442-01 B/LB-DEG R	ENERGY ADDITION PER POUND	8.2632+03 B/LB
FLOW PER POWER	1.2765-04 LB/SEC-KW	OPTIMUM THRUST	5.1210-02 LB
WEIGHT FLOW RATE	7.5950-05 LB/SEC	POWER	5.9500-01 KW

DISTANCE FROM THROAT IN	EFFECTIVE AREA RATIO	PRESSURE RATIO	BOUNDARY LAYER THICKNESS IN	ADJUSTED RADIUS IN	OVERALL EFFICIENCY	HEAT TRANSFER Q B/SEC	EFFECTIVE SPECIFIC IMPULSE SEC	ACTUAL THRUST LB	MACH NUMBER
5.0000-03	1.3024+00	4.6646+00	6.6128-04	1.2933-02	5.4775-01	1.4293-03	4.6772+02	3.5523-02	1.6649+00
10.0000-03	1.5785+00	6.7806+00	1.2004-03	1.4452-02	6.1730-01	2.3092-03	4.9653+02	3.7711-02	1.9051+00
1.5000-02	1.8115+00	8.7194+00	1.6873-03	1.5674-02	6.6943-01	2.9881-03	5.1707+02	3.9271-02	2.0634+00
2.0000-02	2.0215+00	1.0582+01	2.1442-03	1.6736-02	7.0097-01	3.5586-03	5.2911+02	4.0186-02	2.1845+00
2.5000-02	2.2167+00	1.2405+01	2.5813-03	1.7696-02	7.2326-01	4.0596-03	5.3745+02	4.0820-02	2.2839+00
3.0000-02	2.4011+00	1.4208+01	3.0044-03	1.8583-02	7.4030-01	4.5116-03	5.4375+02	4.1297-02	2.3689+00
3.5000-02	2.5775+00	1.6021+01	3.4168-03	1.9414-02	7.5397-01	4.9269-03	5.4874+02	4.1677-02	2.4435+00
4.0000-02	2.7475+00	1.7789+01	3.8209-03	2.0201-02	7.6530-01	5.3133-03	5.5285+02	4.1989-02	2.5102+00
4.5000-02	2.9122+00	1.9576+01	4.2185-03	2.0952-02	7.7492-01	5.6763-03	5.5632+02	4.2252-02	2.5708+00
5.0000-02	3.0725+00	2.1366+01	4.6107-03	2.1673-02	7.8323-01	6.0198-03	5.5929+02	4.2478-02	2.6263+00
5.5000-02	3.2290+00	2.3161+01	4.9984-03	2.2369-02	7.9052-01	6.3469-03	5.6189+02	4.2675-02	2.6777+00
6.0000-02	3.3822+00	2.4961+01	5.3824-03	2.3043-02	7.9699-01	6.9726-03	5.6418+02	4.2850-02	2.7256+00
7.0000-02	3.6803+00	2.8583+01	6.1387-03	2.4333-02	8.0758-01	7.5520-03	5.6792+02	4.3133-02	2.8129+00
8.0000-02	3.9692+00	3.2237+01	6.8862-03	2.5563-02	8.1676-01	8.0946-03	5.7114+02	4.3378-02	2.8909+00
9.0000-02	4.2507+00	3.5927+01	7.6270-03	2.6744-02	8.2446-01	9.1197-03	5.7382+02	4.3582-02	2.9618+00
1.1000-01	4.7955+00	4.3417+01	9.0908-03	2.8987-02	8.3640-01	1.0052-02	5.7796+02	4.3896-02	3.0869+00
1.3000-01	5.3216+00	5.1059+01	1.0544-02	3.1112-02	8.4593-01	1.1777-02	5.8125+02	4.4146-02	3.1955+00
1.7000-01	6.3328+00	6.6792+01	1.3430-02	3.5109-02	8.5921-01	1.3688-02	5.8579+02	4.4491-02	3.3787+00
2.2000-01	7.5426+00	8.7265+01	1.7034-02	3.9785-02	8.7025-01	1.5395-02	5.8954+02	4.4776-02	3.5655+00
2.7000-01	8.7105+00	1.0858+02	2.0649-02	4.4227-02	8.7752-01	1.6960-02	5.9200+02	4.4962-02	3.7217+00
3.2000-01	9.8478+00	1.3067+02	2.4283-02	4.8505-02	8.8237-01	1.8420-02	5.9363+02	4.5086-02	3.8567+00
3.7000-01	1.0961+01	1.5349+02	2.7939-02	5.2662-02	8.8557-01	1.9797-02	5.9471+02	4.5168-02	3.9763+00
4.2000-01	1.2056+01	1.7700+02	3.1618-02	5.6729-02	8.8762-01	2.1109-02	5.9540+02	4.5220-02	4.0837+00
4.7000-01	1.3135+01	2.0116+02	3.5323-02	6.0725-02	8.8881-01	2.2365-02	5.9580+02	4.5251-02	4.1817+00
5.2000-01	1.4200+01	2.2593+02	3.9053-02	6.4664-02	8.8933-01	2.3575-02	5.9597+02	4.5264-02	4.2717+00
5.7000-01	1.5254+01	2.5129+02	4.2808-02	6.8558-02	8.8931-01	2.4746-02	5.9597+02	4.5264-02	4.3553+00

NOZZLE DESIGN OPTIMIZATION RESULTS (cont'd)

NOZZLE NUMBER 99 PROPELLANT HYDROGEN

DESIGN SPECIFICATIONS

NOZZLE DATA

THROAT RADIUS	1.1100-02 IN	IDEAL EXIT RADIUS	4.9630-02 IN
IDEAL LENGTH	8.0000-01 IN	MAXIMUM DIVERGENT ANGLE	2.7000+01 DEG
CONVERGENT RADIUS	1.5540-02 IN	DIVERGENT RADIUS	4.4400-03 IN
WALL TEMPERATURE	2.5000+03 DEG R	THROAT AREA	3.8708-04 IN2

THERMODYNAMIC DATA

STAGNATION PRESSURE	5.0000+00 ATM	AMBIENT PRESSURE	1.0000+00 MM HG
STAGNATION TEMPERATURE	2.7000+03 DEG R	AMBIENT TEMPERATURE	5.4000+02 DEG R
FRACTION DISSOCIATED	4.4025-06	FRACTION IONIZED	4.4721-16
ENTHALPY	1.5694+04 B/LB	BASE ENTHALPY	7.4308+03 B/LB
FLOW PER UNIT THROAT AREA	1.9621-01 LB/SEC-IN2	THROAT TEMPERATURE	2.2940+03 DEG R
IDEAL SPECIFIC IMPULSE	6.7426+02 SEC	THERMODYNAMIC EFFICIENCY	1.1383+00
SPECIFIC GAS CONSTANT	9.8442-01 B/LB-DEG R	ENERGY ADDITION PER POUND	8.2632+03 B/LB
FLOW PER POWER	1.2765-04 LB/SEC-KW	OPTIMUM THRUST	5.1210-02 LB
WEIGHT FLOW RATE	7.5950-05 LB/SEC	POWER	5.9500-01 KW

DISTANCE FROM THROAT IN	EFFECTIVE AREA RATIO	PRESSURE RATIO	BOUNDARY LAYER THICKNESS IN	ADJUSTED RADIUS IN	OVERALL EFFICIENCY	HEAT TRANSFER Q B/SEC	EFFECTIVE SPECIFIC IMPULSE SEC	ACTUAL THRUST LB	MACH NUMBER
5.0000-03	1.3205+00	4.7970+00	6.6877-04	1.3025-02	5.4234-01	1.4233-03	4.6540+02	3.5347-02	1.6832+00
10.0000-03	1.6076+00	7.0147+00	1.2141-03	1.4588-02	6.1701-01	2.3014-03	4.9641+02	3.7702-02	1.9266+00
1.5000-02	1.8464+00	9.0210+00	1.7066-03	1.5828-02	6.7168-01	2.9793-03	5.1793+02	3.9337-02	2.0847+00
2.0000-02	2.0603+00	1.0937+01	2.1684-03	1.6901-02	7.0373-01	3.5489-03	5.3015+02	4.0265-02	2.2051+00
2.5000-02	2.2583+00	1.2806+01	2.6099-03	1.7868-02	7.2608-01	4.0493-03	5.3850+02	4.0899-02	2.3038+00
3.0000-02	2.4451+00	1.4649+01	3.0370-03	1.8760-02	7.4305-01	4.5007-03	5.4476+02	4.1374-02	2.3880+00
3.5000-02	2.6234+00	1.6477+01	3.4532-03	1.9594-02	7.5661-01	4.9154-03	5.4970+02	4.1750-02	2.4620+00
4.0000-02	2.7950+00	1.8298+01	3.8609-03	2.0384-02	7.6781-01	5.3012-03	5.5376+02	4.2058-02	2.5281+00
4.5000-02	2.9610+00	2.0116+01	4.2618-03	2.1137-02	7.7731-01	5.6637-03	5.5717+02	4.2317-02	2.5880+00
5.0000-02	3.1225+00	2.1935+01	4.6572-03	2.1860-02	7.8550-01	6.0067-03	5.6010+02	4.2540-02	2.6430+00
5.5000-02	3.2800+00	2.3756+01	5.0479-03	2.2556-02	7.9268-01	6.3333-03	5.6266+02	4.2734-02	2.6939+00
6.0000-02	3.4341+00	2.5581+01	5.4347-03	2.3231-02	7.9904-01	6.6457-03	5.6491+02	4.2905-02	2.7414+00
6.5000-02	3.5852+00	2.7411+01	5.8182-03	2.3886-02	8.0473-01	7.2458-03	5.6691+02	4.3057-02	2.7858+00
[illegible].5000-02	3.8798+00	3.1092+01	6.5746-03	2.5145-02	8.1415-01	7.8048-03	5.7022+02	4.3309-02	2.8674+00
8.5000-02	4.16[illegible]9+00	3.4802+01	7.3231-03	2.6350-02	8.2239-01	8.3307-03	5.7310+02	4.3527-02	2.9409+00
9.5000-02	4.4448+00	3.8545+01	8.0654-03	2.7509-02	8.2937-01	9.3278-03	5.7553+02	4.3711-02	3.0080+00
1.1500-01	4.9856+00	4.6123+01	9.5329-03	2.9719-02	8.4028-01	1.0239-02	5.7930+02	4.3998-02	3.1274+00
1.3500-01	5.5083+00	5.3862+01	1.0990-02	3.1818-02	8.4906-01	1.1932-02	5.8232+02	4.4227-02	3.2316+00
1.7500-01	6.5137+00	6.9743+01	1.3884-02	3.5775-02	8.6140-01	1.3815-02	5.8654+02	4.4547-02	3.4086+00
2.2500-01	7.7172+00	9.0357+01	1.7497-02	4.0415-02	8.7172-01	1.5503-02	5.9004+02	4.4814-02	3.5901+00
2.7500-01	8.8791+00	1.1177+02	2.1119-02	4.4829-02	8.7854-01	1.7055-02	5.9234+02	4.4988-02	3.7426+00
3.2500-01	1.0010+01	1.3393+02	2.4757-02	4.9083-02	8.8307-01	1.8503-02	5.9387+02	4.5104-02	3.8749+00
3.7500-01	1.1118+01	1.5679+02	2.8417-02	5.3221-02	8.8604-01	1.9872-02	5.9487+02	4.5180-02	3.9922+00
4.2500-01	1.2206+01	1.8031+02	3.2098-02	5.7269-02	8.8792-01	2.1176-02	5.9550+02	4.5228-02	4.0979+00
4.7500-01	1.3279+01	2.0446+02	3.5833-02	6.1247-02	8.8898-01	2.2426-02	5.9585+02	4.5255-02	4.1942+00
5.2500-01	1.4338+01	2.2920+02	3.9532-02	6.5169-02	8.8939-01	2.3631-02	5.9599+02	4.5265-02	4.2830+00
5.7500-01	1.5386+01	2.5451+02	4.3285-02	6.9047-02	8.8929-01	2.4797-02	5.9596+02	4.5263-02	4.3653+00

NOZZLE DESIGN OPTIMIZATION RESULTS (cont'd)

NOZZLE NUMBER 59 PROPELLANT HYDROGEN

DESIGN SPECIFICATIONS

NOZZLE DATA

THROAT RADIUS	1.1100-02 IN	IDEAL EXIT RADIUS	4.9630-02 IN
IDEAL LENGTH	8.0000-01 IN	MAXIMUM DIVERGENT ANGLE	2.9000+01 DEG
CONVERGENT RADIUS	1.5540-02 IN	DIVERGENT RADIUS	4.4400-03 IN
WALL TEMPERATURE	2.5000+03 DEG R	THROAT AREA	3.8708-04 IN2

THERMODYNAMIC DATA

STAGNATION PRESSURE	5.0000+00 ATM	AMBIENT PRESSURE	1.0000+00 MM HG
STAGNATION TEMPERATURE	2.7000+03 DEG R	AMBIENT TEMPERATURE	5.4000+02 DEG R
FRACTION DISSOCIATED	4.4025-06	FRACTION IONIZED	4.4721-16
ENTHALPY	1.5694+04 B/LB	BASE ENTHALPY	7.4308+03 B/LB
FLOW PER UNIT THROAT AREA	1.9621-01 LB/SEC-IN2	THROAT TEMPERATURE	2.2940+03 DEG R
IDEAL SPECIFIC IMPULSE	6.7426+02 SEC	THERMODYNAMIC EFFICIENCY	1.1383+00
SPECIFIC GAS CONSTANT	9.8442-01 B/LB-DEG R	ENERGY ADDITION PER POUND	8.2632+03 B/LB
FLOW PER POWER	1.2765-04 LB/SEC-KW	OPTIMUM THRUST	5.1210-02 LB
WEIGHT FLOW RATE	7.5950-05 LB/SEC	POWER	5.9500-01 KW

DISTANCE FROM THROAT IN	EFFECTIVE AREA RATIO	PRESSURE RATIO	BOUNDARY LAYER THICKNESS IN	ADJUSTED RADIUS IN	OVERALL EFFICIENCY	HEAT TRANSFER Q B/SEC	EFFECTIVE SPECIFIC IMPULSE SEC	ACTUAL THRUST LB	MACH NUMBER
5.0000-03	1.3384+00	4.9291+00	6.7609-04	1.3115-02	5.3656-01	1.4177-03	4.6292+02	3.5158-02	1.7009+00
10.0000-03	1.6359+00	7.2451+00	1.2274-03	1.4719-02	6.1665-01	2.2941-03	4.9626+02	3.7691-02	1.9469+00
1.5000-02	1.8800+00	9.3147+00	1.7250-03	1.5976-02	6.7383-01	2.9709-03	5.1876+02	3.9400-02	2.1047+00
2.0000-02	2.0974+00	1.1280+01	2.1914-03	1.7058-02	7.0634-01	3.5398-03	5.3113+02	4.0339-02	2.2245+00
2.5000-02	2.2980+00	1.3192+01	2.6371-03	1.8031-02	7.2873-01	4.0395-03	5.3948+02	4.0974-02	2.3224+00
3.0000-02	2.4869+00	1.5071+01	3.0680-03	1.8926-02	7.4562-01	4.4903-03	5.4570+02	4.1446-02	2.4059+00
3.5000-02	2.6669+00	1.6933+01	3.4878-03	1.9763-02	7.5906-01	4.9045-03	5.5060+02	4.1818-02	2.4791+00
4.0000-02	2.8398+00	1.8784+01	3.8988-03	2.0555-02	7.7015-01	5.2898-03	5.5460+02	4.2122-02	2.5446+00
4.5000-02	3.0071+00	2.0630+01	4.3027-03	2.1310-02	7.7952-01	5.6517-03	5.5797+02	4.2378-02	2.6040+00
5.0000-02	3.1696+00	2.2474+01	4.7010-03	2.2034-02	7.8760-01	5.9943-03	5.6085+02	4.2597-02	2.6585+00
5.5000-02	3.3280+00	2.4320+01	5.0944-03	2.2731-02	7.9467-01	6.3204-03	5.6336+02	4.2787-02	2.7089+00
6.0000-02	3.4829+00	2.6167+01	5.4838-03	2.3407-02	8.0093-01	6.6323-03	5.6558+02	4.2956-02	2.7559+00
6.5000-02	3.6347+00	2.8019+01	5.8698-03	2.4062-02	8.0653-01	7.2315-03	5.6755+02	4.3105-02	2.8000+00
7.5000-02	3.9304+00	3.1739+01	6.6308-03	2.5322-02	8.1579-01	7.7897-03	5.7080+02	4.3352-02	2.8808+00
8.5000-02	4.2174+00	3.5484+01	7.3833-03	2.6527-02	8.2388-01	8.3148-03	5.7362+02	4.3567-02	2.9536+00
9.5000-02	4.4970+00	3.9258+01	8.1294-03	2.7686-02	8.3074-01	9.3104-03	5.7600+02	4.3747-02	3.0201+00
1.1500-01	5.0387+00	4.6901+01	9.6035-03	2.9895-02	8.4144-01	1.0220-02	5.7970+02	4.4028-02	3.1384+00
1.3500-01	5.5618+00	5.4675+01	1.1066-02	3.1992-02	8.5007-01	1.1911-02	5.8267+02	4.4254-02	3.2418+00
1.7500-01	6.5671+00	7.0621+01	1.3968-02	3.5944-02	8.6216-01	1.3791-02	5.8680+02	4.4567-02	3.4173+00
2.2500-01	7.7691+00	9.1283+01	1.7588-02	4.0577-02	8.7227-01	1.5477-02	5.9023+02	4.4828-02	3.5974+00
2.7500-01	8.9286+00	1.1271+02	2.1215-02	4.4983-02	8.7894-01	1.7027-02	5.9248+02	4.4999-02	3.7487+00
3.2500-01	1.0057+01	1.3486+02	2.4857-02	4.9228-02	8.8336-01	1.8474-02	5.9397+02	4.5112-02	3.8801+00
3.7500-01	1.1161+01	1.5769+02	2.8517-02	5.3356-02	8.8625-01	1.9842-02	5.9494+02	4.5186-02	3.9965+00
4.2500-01	1.2245+01	1.8117+02	3.2198-02	5.7394-02	8.8806-01	2.1145-02	5.9555+02	4.5232-02	4.1015+00
4.7500-01	1.3313+01	2.0525+02	3.5901-02	6.1361-02	8.8906-01	2.2394-02	5.9588+02	4.5257-02	4.1972+00
5.2500-01	1.4368+01	2.2990+02	3.9627-02	6.5273-02	8.8943-01	2.3597-02	5.9600+02	4.5266-02	4.2854+00
5.7500-01	1.5410+01	2.5512+02	4.3376-02	6.9139-02	8.8929-01	2.4762-02	5.9596+02	4.5263-02	4.3672+00

APPENDIX B

B. References

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